THE
MATHEMATICAL GAZETTE.

EDITED BY
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MATHEMATICAL ASSOCIATION.

REPORT OF THE COUNCIL FOR 1909.

THE Council deeply regret to have to record that, during the past year the Association has lost by the death, on 12th July, of SIMON NEWCOMB, Emeritus Professor of Johns Hopkins University, one of its most distinguished Honorary Members; by the death of C. A. RUMSEY, of Dulwich College, on 2nd September, one who was for some years, and until last January, a valued member of the Council; and by the death of J. W. WELSFORD, of Harrow, on 29th April, one who took an active part in the work of the original Association in its earlier days.

Since the last annual meeting 112 new members have been elected, and the Association now consists of 548 members. Of these 8 are honorary, 29 are life members by composition, 52 are life members under the rule which has now been rescinded, and 459 are ordinary members.

Until recently the Library of the Association was not in possession of copies of the first six numbers of the *Gazette*, which were issued in quarto form in 1894, but that want has now been supplied by the kindness of the family of the late Mr. GEORGE HEPPEL, who have given to the Library a complete set of these numbers and also a few duplicates. With these duplicates and additional copies obtained from another source, the Librarian has been able to form a second complete set of the numbers 1 to 6. The Rev. J. J. Milne has generously presented to the Library a complete set of the 19 annual reports of the Association for the Improvement of Geometrical Teaching, from which this Association sprang, and some other works which will be useful additions to the Library.

The Joint Committee of the Mathematical Association and the Association of Public School Science Masters have been at work during the year and have issued a report on the Correlation of Mathematical and Science Teaching, which was published in the *Mathematical Gazette*, No. 82. The Report is issued also in pamphlet form, and can be obtained from the publishers, Messrs. G. Bell & Sons, Ltd., at sixpence per copy.

The Council have had under consideration during the year the question of the formation of local branches of the Association, and rules by which the connection between a local branch and the Association should be regulated. The North Wales Branch at Bangor has been in existence for some time: a desire has been expressed that a branch should be formed in London: and the Southampton Mathematical Society has signified its wish to be affiliated to the Association. The need of suitable regulations had therefore to be met, and the result of the Council's deliberations was published in the *Mathematical Gazette*, No. 81. Criticism, and suggestions for amendment, of these draft regulations were invited, and the Association at this meeting will be asked to approve formally such regulations as may be deemed fitting, and to sanction such alterations in the laws of the Association as may thereby be rendered necessary.

A sub-committee appointed by the Council to proceed with the preliminaries to the formation of a London branch held a meeting, at which Professor W. H. H. Hudson presided, at the Polytechnic, Regent Street, on 27th November. The meeting was largely attended, and the proposal that a London branch should be formed was warmly welcomed and approved.

The Council nominated Mr. D. B. Mair as the representative of the Association at the Conference of the Delegates of Scientific Societies (under the direction of the British Association) held at Burlington House on 25th and 26th October.

The Association was invited to send two delegates to act on the General Committee of the North of England Education Conference held at Leeds on the 6th to the 10th of January, 1910, and Professor A. Leahy of Sheffield University and Mr. A. O. Allen of Leeds University accepted the Council's nomination.

The Council regret that Miss Greene has found it necessary to resign her office as one of the Honorary Secretaries of the Association. The Association is asked to appoint her successor at this meeting. In the brief period during which Miss Greene has held office the Association has largely benefited by the energy with which she has made the objects of the Association known to the large number of ladies who are interested in the teaching of elementary mathematics. Miss Greene has done a great amount of work very successfully, and in a very short time.

Professor W. H. Wagstaff and Mr. C. C. Lynam now retire from the Council in compliance with the rules, and this meeting is asked to elect two other members to take their places.

The Council desire to nominate Dr. H. T. Bovey, late director of the Imperial College of Science and Technology, as an Honorary Member of the Association.

The Council desire again to express their cordial appreciation of the services of Mr. W. J. Greenstreet as editor of the *Mathematical Gazette*, and their thanks also to the Council of King's College for their kindness in affording them all the accommodation which they have needed for their meetings.

ANNUAL MEETING.

THE Annual Meeting of the Association was held in the Library of Westminster School at 10.30 a.m. on Wednesday, 12th January, 1910, the President, Professor H. H. Turner, D.Sc., F.R.S., in the chair.

Mr. C. Pendlebury read the Report of the Council for the year 1909, Mr. F. W. Hill made his financial report, and the reports were adopted.

Dr. H. T. Bovey, late Director of the Imperial College of Science and Technology, was elected an Honorary Member of the Association.

Miss E. R. Gwatkin was elected an Honorary Secretary in place of Miss E. Greene; Miss Greene and Dr. T. P. Nunn were elected members of the Council in place of Mr. C. C. Lynam and Professor W. H. Wagstaff.

A very cordial vote of thanks was accorded to Miss Greene for her services to the Association as an Honorary Secretary during the past two years.

The meeting gave its sanction to the formation of local branches and to the regulations which had been drawn up by the Council, and approved of the consequent alterations in the Rules of the Association.

The Committee on the Teaching of Elementary Mathematics was re-appointed.

On the conclusion of the business part of the meeting the President gave an address, and Mr. C. Godfrey read a paper on "The Teaching of Algebra; what is Educational and what is Technical?" Mr. Godfrey's paper was followed by a discussion in which Professor Alfred Lodge, Dr. F. S. Macaulay, Mr. P. Abbott, Dr. T. P. Nunn, Miss B. Hewett, Mr. W. N. Roseveare, Mr. J. V. H. Coates, Mr. E. H. Butt, Mr. C. H. Richards, and Prof. H. H. Turner took part.

It was much regretted that time did not allow Professor P. J. Harding and Rev. J. J. Milne to give more than very brief

outlines of the papers which they had offered; and that the reading of Mr. C. V. Durell's paper had to be abandoned.

PRESIDENTIAL ADDRESS.

BY PROFESSOR H. H. TURNER.

IN acknowledging with mingled feelings of gratitude and anxiety the great honour you did me in electing me your President, I may perhaps venture a word or two in explanation of the circumstance that this honour was conferred in my absence. I should have attended the annual meeting of last year but for an engagement of some pressing importance in Egypt. A large reflecting telescope was presented to the Egyptian Government in 1905 by Mr. J. H. Reynolds of Birmingham, and the arrangements for its installation and working having been completed after some unavoidable delays, the Government did me the honour to invite me to inspect the instrument and report on its prospects. By using the Christmas vacation I was just able to spend a sufficient time witnessing the instrument at work in the able hands of Mr. Knox Shaw, and to discuss the programme of work with him and with Mr. Keeling, the Director of the Helwan Observatory. My task was not difficult, consisting mainly in the approval of what had already been done; and the instrument, observer and climate have recently given a striking proof of their combined efficiency by photographing Halley's Comet on August 24, more than a fortnight before any other successful photograph was taken. I trust this happy inception presages a long career of usefulness and success for the instrument.

But I might not have troubled the Association with this personal apology did it not lead up to a matter which is of more direct interest to us, as I hope to show presently. My visit to Egypt enabled me to see something of the dramatic manner in which a reign of scientific efficiency has been established in Egypt by Captain H. G. Lyons, late R.E., who has now accepted an appointment at Glasgow University, which offers opportunities for scientific work, and more freedom from the heavy administrative work, which was beginning to absorb his time too completely. But before leaving Egypt he put the Survey and its various ramifications on a secure basis, so that he could with confidence leave the future development of this noble work to the staff of brilliant young men whom he had collected round him. It is to the manner in which he selected this staff that I wish to draw attention in a moment, but I must first say a word or two more about the work itself, which may be described without exaggeration as the evolution of order out of chaos, and that with a

rapidity that was almost startling. As an illustration of the original chaos I cannot do better than quote an incident which falls within my own department of astronomy, though it must be remarked that this is only a small part of the varied work of the Survey. Captain Lyons found a time system in full working order: a gun was fired at noon every day from the observatory, and on the face of it this was most satisfactory. But to make sure of the essential facts he paid a visit to the observatory, then in Cairo, and asked to see the transit circle. His question was apparently not understood, and so he asked the officer in charge how he knew when to fire the gun. The officer produced a watch which told him the time. "But how do you know your watch keeps correct time?" was the remorseless question. "Oh! I take it to the watchmaker in Cairo once a week and he tells me the error." So Captain Lyons went to the watchmaker named and asked him whether it was true that he supplied the error of the watch; it turned out to be so far true. "And how do you get correct time?" "Oh! I get that from the gun!"

This single example will sufficiently illustrate the working of the Oriental mind. Captain Lyons unravelled its workings in other connections, such as the meaning of north and south, which to a dweller by the Nile generally mean down and up stream, wherever the sun may be; and again in the recording of the multitudinous tiny plots of land which he had to survey for revenue purposes—records made with scrupulous care in figures, as they had been made for thousands of years, but without drawing any map, so that when a map was drawn it was seen at once that some portions were counted twice over and others not at all. Starting from such unpromising beginnings, he has completed an admirable survey of the country and made maps of it. These maps he displayed in an exhibition in Cairo, which was visited by thousands of landowners who had never seen their property represented in this way before. He has initiated a magnetic survey and other valuable magnetic work: he has organised meteorological work (so vitally important for Egypt, which lives or dies by the vagaries of the rainfall in Abyssinia, the source of the Nile flood), including the most modern researches on the upper air: he measures and reports upon the volume of the flood; and realising how important for the future of Egypt are all questions of precise level which may affect the flow of the Nile, he seized an opportunity of undertaking a great piece of survey work of this nature, which is already bearing excellent fruit.

Now, it will be readily imagined that for work of such extent and variety it is not easy to get a suitable staff of assistants. Scientific knowledge is necessary, but so also is a knowledge of

Arabic and a physique which will stand the hot climate; so also is a business capacity and a faculty of detecting the truth in its Oriental disguises. It might well be that any one of these qualities was essential, while the rest, though desirable, might have to be dispensed with; or it might be that some rare combination of them must be sought with toil. It will probably be admitted that the final opinion of a man who has gone through the trying experience of getting together a staff suitable for such work, and finds himself ultimately satisfied as to the right course, is worth hearing; and hence I feel that the Association will learn with peculiar pleasure that Captain Lyons's final method is to take able mathematicians from Cambridge or Oxford and trust to luck for the other qualities. The one thing he finds needful is that, when some strange situation occurs, they should have a firm grasp of the fundamental principles and not merely a knowledge of the rules deduced, which may fail to meet some unforeseen contingency. And this essential condition Captain Lyons has found to be fulfilled by mathematicians when others have failed to meet it. His faith in them has been justified in cases where a breakdown might have possibly been admitted. Even the most complete knowledge of mathematical or physical principles could scarcely be expected to inspire a man in dealing with an Arab camel driver who was shamming sick; or with the organisation of the commissariat for a journey in the desert; or with an unexpected attack by wandering tribes which necessitated addressing them with dignity from the hump of a camel with three rifles pointed at one's chest. But it has been proved in the best possible way, viz. by actual experience, that such situations are dealt with capably by young men selected for their mathematical ability, with no special training for the contingencies of life beyond what undergraduates all pick up from life in one of our great universities. This is a lesson which we may well lay to heart. It is not entirely new, at any rate to astronomers. The Chief Assistants at the Royal Observatory at Greenwich have been selected on this plan for seventy years past, and I hope it does not savour of vanity to say that I think it has worked well. But it is only too easy to think of reasons why it might have failed disastrously. If anyone cares to study the success of it more closely, I can heartily recommend him to find some good pretext for visiting Egypt (and the scientific opportunities are so great as to offer many such pretexts) and for spending some days at the Helwan Observatory. There he will find a hearty welcome from a constantly changing little community, who will rig him up a tent and assign him a place at their table; he will hear of their varied experiences, perhaps see one of them depart on a new enterprise, or be present at the return of another bringing such news as the desert can furnish:

a little community of which every member is taught from the first to rely on no intelligence but his own for solving any of the difficulties of life, and most of whom have had at least one hard nut to crack, and have cracked it.

In thus singing the praises of a mathematical education I am well aware of several objections that may be made to my inferences. It may be remarked, for instance, that the surveying work to be done in Egypt is of a special kind for which a mathematical education is specially adapted. To this I have already replied in part by pointing out that the work is much more varied than might be supposed. Not only are the technical problems quite new to those who deal with them, but it is an essential requisite of the work to learn a new language and to deal with a native race: the work involves, in fact, much that is needed for government in general. It may also be urged that our universities inevitably provide, besides a special training in mathematics, a general education of a wider kind. To this I cheerfully assent, provided that enough stress is laid on the association of man with man, and not too much on the compulsory acquisition of an ignorance of Greek. The proper general education is largely provided by the men themselves who are thrown together, and is independent of the curriculum when it is not hampered by it. Finally, I am ready to give ample weight to the wisdom of Captain Lyons in allowing newly arrived members of his staff six months in which to learn Arabic and become accustomed to the new conditions, and also to the educative force of responsibility. There is no better way of learning to do a thing than doing it. But when all this is said, I still think there remains enough of valuable testimony to the excellence of a mathematical education, in the usual sense in which it is attacked as too exclusively specialist.

There is a famous attack on Mathematics by Sir William Hamilton, which first appeared in the *Edinburgh Review* for January, 1836, but was reprinted as late as 1853 with but slight modifications, so that we may take it to have represented the deliberate judgments of its author. It was called forth by a small pamphlet of Whewell's entitled *Thoughts on the Study of Mathematics as a Part of a Liberal Education*, in which Whewell was chiefly concerned "to urge the introduction of Mechanics and Hydrostatics into all examinations for the Bachelor of Arts' degree," these sciences being "excellent examples of that great system of physical knowledge which has been steadily advancing ever since the revival of learning in Europe." In view of the particular nature of his plea, Whewell claimed that Hamilton's thunderbolt was aimed wide, and did not touch his pamphlet, though he added that he could give Hamilton as good as he gave, if necessary. Whether he ever carried out this threat

I do not know. At any rate it is not my present purpose to give an account of the controversy, though I do wish to glance at one or two points of interest among many that are suggested by the two papers quoted.

Hamilton's main conclusions are strongly adverse to the study of mathematics. "Of all our intellectual pursuits, the study of the mathematical sciences is the one whose utility as an intellectual exercise, when carried beyond a moderate extent, has been most peremptorily denied by the greatest number of the most competent judges: and the arguments on which this opinion is established have hitherto been evaded rather than opposed."

This statement occurs near the outset, but it represents what he means to prove, and is reiterated in various forms throughout his able and interesting essay.

"The Cambridge Catholicon," he writes in another place, "is thus a dose which never bestows health, but tends always to evolve the seeds of disease."

It is not my intention to oppose his arguments, which seem to me very good. But I do not accept his conclusions: the existence of the little community of mathematicians to which I have just referred seems to me a very good reason for not accepting these conclusions; and it would not need a prolonged search to discover other equally good concrete examples of the falsity of such conclusions. Now, what do we do, whether we be philosophers or mathematicians, when we get a result at variance with known facts, supported by what is apparently a sound piece of reasoning? If we find no flaw in the working, we are thrown back on the fundamental assumptions made at the outset, and must examine them with new care. So in the case of Hamilton's conclusions, finding them at variance with my experience, and paying him the compliment of supposing that he has done the arithmetic correctly, I naturally look at his original assumptions, and I find at least one of them which I cannot admit.

"The end of a liberal education," he assumes, "is the general and harmonious evolution of [the mind's] faculties and capacities in their relative subordination."

There is here a tacit implication that minds are all of one type, with a determinate relation between the various faculties and capacities. I sincerely trust that this conception is now an anachronism, but at the time when Hamilton wrote we must do him the justice to remember that Whewell, whom he was attacking, would probably have accepted the above statement: for, as we have seen, Whewell was urging "the introduction of Mechanics and Hydrostatics into *all* examinations for the Bachelor of Arts' degree," not merely into those for mathematicians. Those were still the days of universal rules—*e.g.* everyone must take the

mathematical tripos—and this tyranny resulted from the conception that human minds were essentially of one type, with the corollary that they should all be trained alike; that a "general education" could be devised suitable for all. This is the insidious half-truth which has done, and is still doing, so much harm in education. Complete the truth according to the well-known formula that a man should know "something of everything and everything of something," and we get a precept which may or may not be valuable, but is at least harmless. But the one half is an antidote to the other, and either by itself is a veritable poison. Let us grant at once that a mere specialist is a grave educational mistake: do we not get as serious mistakes from striving after what is called a "general education"? Such persistent striving has at any rate landed us in one absurd situation, which is familiar to many here. A boy has just proved his mathematical ability by obtaining a scholarship, and has a term or two still at school which might be spent in leisurely adding some clothing to the rather dry bones of teaching which the pressure of a scholarship examination imposes. But instead of this he must cram Greek for the purposes of an entrance examination, because this poisonous notion of a "general education" still infects our educational system. Stories of "forcible feeding" such as have been recently reported to our unwilling ears no doubt shock our sense of decency: but that "forcible feeding" has at any rate the laudable aim of saving life, while compulsory Greek is only crammed for the loathsome purpose of being ejected in examination. I do not forget that mathematics has been, and still is to some extent, forcibly imposed on unwilling victims: nor must I omit to mention an anxiety which has lately come upon us, lest certain well-meaning but misguided people should endeavour to establish compulsory science. But I think mathematicians have realized the injustice they were doing, and taken pains to set it right earlier than literary specialists: and I sincerely trust that the representations of those chiefly concerned will suffice to prevent the scientific danger. Compulsory Greek is the greatest surviving outrage upon decency which results from the striving after a "general education" for all boys—treating them all alike until they are presumed to be of a proper age to specialize. A gardener might as well treat all his flowers as cabbages throughout the most important period of their development. We have too long shut our eyes to the fact that boys are born different and diverge in character with every additional week of growth. To apply the same formula to them all may be a necessity of early experiments in education, but must yield to the advance of civilization. That the time has come for a vigorous attempt to combat the evil demon which goes by the name of a "general education" there are many signs.

His greatest stronghold is the University of Oxford, where I have the honour to reside alongside him. A short time ago "Some Oxford Tutors" wrote a series of letters to *The Times*, in which they showed a commendable desire to reform certain abuses and spoke much good sense: but they also made it plain that they were slaves to the demon by proposing to give him power over the whole undergraduate course, relegating to the post-graduate period all "specialist" studies, such as chemistry or astronomy. Oxford annually sends out numbers of young men, trained "generally" by the demon, who have most agreeable manners and are ready to govern the country or manage the press, in the cheerful conviction that if they should happen to want expert opinion or advice they can get it by asking: though it is usually easier not to ask. We are familiar with some of the consequences of this system: it is a system which leads, for instance, to the appointment of special commissions for investigating tropical diseases, but hampers or prevents entirely any action necessary for taking advantage of the important results obtained. The specialist is constantly mistrusted by the man of "general education," whose knowledge of anything has never exceeded a modest respectability: and the only remedy for this state of affairs is to have more specialists among our governing classes. Special knowledge is urgently needed in all directions: let me take another instance that occurs as I write. In *The Times* of Jan. 10 is an account of Professor Omori's report on the Messina earthquake, in which he gives it as his opinion, after a three months' investigation on the spot, that "about 998 out of 1000 of the number killed in Messina must be regarded, when considered in comparison with a Japanese city, as having fallen victims to the seismologically bad construction of the houses." The Japanese have been quick to protect themselves in this way, and they learnt how to do it from Englishmen, and the Englishmen who taught the Japanese were not men of "general education," but men who knew one thing well. I doubt if any one of them was an Oxford man. It seems to be a consequence of the subordination of the special to the general in Oxford that in many branches of knowledge no specialists ripen there, so that in Oxford there are several aliens like myself who owe thanks to the "general education" demon for our annual incomes.

And indeed we owe him much more than that: for to live in Oxford is a great privilege and a great education. Let me not leave in doubt my deep appreciation of the greatness of the University, or the value of its teaching, by which I trust I have personally profited much. It would argue a fatal defect in myself were it not so: and among the many lessons I have learnt, not the least valuable have been those on the proper method of dealing with the evil we are considering. I have been

led to wonder whether our methods have not been hitherto too directly aggressive. I doubt if we shall ever conquer the demon in "pull devil, pull baker" fashion, for his own strength grows with our opposition. The real fact is that there should be no opposition between us, for the two principles of a general and a special education are not mutually destructive but necessary to each other, and it is only when both are living realities that their joint action is truly beneficial. I believe the proper way to deal with the demon is, not to attack him, which seems to make him grow, but simply to swallow him, which will reduce him to his proper dimensions. In other words, why should not mathematicians and men of science devise a "general education" of their own, far more comprehensive than that which at present usurps the name? It would not be difficult, and we have indeed been remiss in not doing it before: but it is not too late to start now. I say we have been remiss hitherto, and a single example will perhaps suffice to make the point clear. Some years ago I was asked by my friend Professor Sampson to examine the University of Durham in mathematics. My duties proved to include an examination in either English History or Logic, whichever I preferred. "Don't say you can't," wrote Professor Sampson, "because we all do it: you won't find it as difficult as you think": and urged in this kindly fashion I made shift to attempt the examination in History. But I was led to ponder on the question, "Why should it have seemed so strange for a mathematician to examine in English History? Is it not as much our property as that of the specialists in Classics? Have we not neglected our just claims too long?" I may be wrong, but I fancy that cases of a mathematical master in a public school also teaching English History are rare. But is this a necessary or even decent state of affairs when one comes to think it over seriously? An essay again, is it not common property? In the University of Oxford, to which I have already referred, many essays are written and read: but entirely by those whose studies have usurped the title "humane": those who, like ourselves, are concerned with matters of more vital importance do not write or read essays. I say boldly that I think we have made a grave mistake: and we have strengthened the hands of our opponents by the error. By stretching the idea of a "general education" too far they have landed us in the absurdities and evils of compulsory Greek: but by equally grave faults in the other direction we have enabled them to reproach us as indifferent to our own language and history, and good will come out of evil if the long struggle for freedom from literary tyranny leads us to recognize this mistake and set it right. There are not wanting signs that we have as a nation erred in this direction beyond other nations: for instance, while on the Continent the

historical sympathies of mathematicians have been sufficiently developed to secure great editions of the works and correspondence of their heroes, we in England have so far omitted to discharge this pious duty for Newton. I am glad to say that the project is under consideration by the Cambridge Philosophical Society: but the tardiness of commencement is an indication of imperfect historical sympathy. Some years ago Professor Sampson and I discussed the question whether the publication of Newton's works and correspondence, being obviously a huge task for one man, might not be better undertaken by a Newton Club, formed for the purpose. We felt that there might be many advantages in such a course, not the least of which would be the quickening of the historical sympathies among those who joined in the project. But we got little encouragement to proceed at the time, though possibly a more favourable moment may come in the future. Such matters are worthy of more than passing attention: for if mathematicians are indifferent to the history of their own subject, and the fame of their own heroes, it is more than likely that their wider sympathies are deficient. An Association of this kind might profitably discuss the magnitude of this evil and its remedy.

But it may seem to many here that prospects of mathematicians teaching history on equal terms with their literary colleagues, or looking over essays, are somewhat Utopian: are at any rate too far removed from the sphere of practical politics at present. Admitting that our limits have been too narrow and ought to be widened, we must begin with something much less ambitious and revolutionary. Such counsels are generally counsels of wisdom; and from their standpoint we may welcome especially the movement which is to receive our attention this afternoon. If our boundaries are to be extended, the most natural and least aggressive method is to make friends with our nearest neighbours, namely our physical and chemical neighbours just over the fence. It is strange indeed that a fence should ever have existed, but there is no doubt of its existence or even of its vitality; for though it has been broken down more than once, it seems to spring up again. Whewell's essay of 1835, which drew the thunderbolts of Hamilton, was an attack upon the fence. As already remarked, it was an attempt "to urge the introduction of Mechanics and Hydrostatics into all examinations for the Bachelor of Arts' degree." He remarked in a letter, commenting upon Hamilton's attack: "All the Reviewer's arguments, and I believe the judgments of all his cloud of witnesses, are founded upon the nature and processes of pure mathematics only; on a consideration of the mere properties of space and number. My suggestion [is that we should] avoid confining ourselves to pure mathematics."

If we may for a moment glance back to an earlier date, a most interesting memorandum of Newton's has just been unearthed and published by Mr. Rouse Ball, in which he gives his views on University Studies. He thinks that each Undergraduate should have a Tutor, a Humanity Lecturer, a Greek Lecturer, a Philosophy Lecturer, and a Mathematic Lecturer. The Philosophy was to be chiefly Natural Philosophy—Motion and its laws, Mechanical Powers, Hydrostatics, System of the World, Meteors, Minerals, Vegetables, Animals—ending with "Anatomy if he have skill therein": and the separation of Philosophy from Mathematic thus corresponds to the separation of Science from it nowadays, though it is to be noted that Mathematic at that time included Astronomy, Optics, and Music, with the principles of Chronology and Geography. We are interested to read that "because the Philosophy and Mathematic Lecturers' office is laborious, for encouraging them to diligence none shall be compelled to come to their lectures, but all that will be auditors shall offer each of them a quarterly gratuity." Newton desired no compulsory Mathematic or Science: it would have been well if he could have had his way.

There is one more sentence that has a special interest for us. He writes: "All students who will be admitted to lectures in Natural Philosophy to learn first Geometry and Mechanics. By Mechanics I mean here the demonstrative doctrine of Forces and Motions, including Hydrostatics. For without a judgment on these things a man can have none on Philosophy." He thus foreshadows the organization of Mathematics and Science in relation to one another, which is our concern at this moment.

Newton had already become anxious that the boundary between Mathematics and Science should be crossed in one direction, viz. that those who were going to study science should first learn some mathematics: the complementary proposition, that it was good for mathematicians to learn to observe and experiment, does not seem to have occurred to him, perhaps because it was too obvious. But Whewell reminds us of the need by his endeavour to get mathematicians to learn mechanics and hydrostatics, which had apparently dropped out of the curriculum since Newton's time. And although much physical learning is now included in the schedule for the mathematical tripos, it is treated in a symbolic manner, without observation or experiment. The fence between mathematics and science is still there. And not only is it in these high places, but it is a well-known boundary in school teaching. There is generally no organized joint action between masters who teach mathematics and masters who teach science: there is often not even a good understanding between them, by which I mean that the methods learnt in one class-room are spoken of in another with some

disrespect. Such cases have come to my own knowledge as occasional inspector of Public School mathematics for the Oxford and Cambridge Joint Board. But I am glad to say that I have also seen something of the admirable beginning that has been made in some schools towards a joint organization of the teaching. The courses of practical mathematics which have been found so successful of recent years have paved the way for further developments. To say more would be to trench on the discussion to which we are looking forward: but I cannot close this address without a word or two of hearty thanks to those who have promoted that discussion, for it is my personal conviction that they have thereby promoted the welfare of mathematics: and I will express an earnest hope that the outcome of the discussion may be something simple in its practical application and far-reaching in its results.

THE TEACHING OF ALGEBRA; WHAT IS EDUCATIONAL AND WHAT IS TECHNICAL?

By Mr. C. Godfrey.

The value of a mathematical training is often discussed with the latent assumption that different parts of mathematics are equally valuable, and have a similar educational effect. Now I should like to suggest that the educational effects of geometry and algebra are as different as the educational effects of Greek and piano-playing.

The precise place of algebra in education deserves, I think, more consideration than is generally given to it. I want to make a distinction between the *ideas* of algebra and the *technique* of algebra.

What are the leading ideas of schoolboy algebra?

One leading idea is the idea of generalisation, the generalisation of an infinity of particular statements into a single universal formula. To take a simple instance: twice seven = seven times two; three times twelve = twelve times three, and so on for ever. We can pack this endless multitude of truths into a sentence by saying that "the result of multiplication is independent of the order in which the two factors are taken." And we can compress the sentence into a formula; $ab=ba$. What could be neater?

There is no end to the series of general truths that find their simplest expression in an algebraic formula; every algebraic identity is short for a sentence; and this sentence contains an infinity of particular statements. An algebraic formula looks unpoetical enough; but to my mind there is a certain aesthetic side to this power of pregnant expression that algebra possesses. It is analogous to Newton's grand generalisation of the move-

ments of the heavenly bodies. Newton took as his problem the infinitely complicated movements of the sun and planets; movements that had been watched for thousands of years before people could disentangle the skein. At the end of his labours he said: "In all these movements of earth, sun, moon and planets, I discern one thing happening, and one only. Every particle in the universe is attracting every other particle with a force varying inversely as the square of the distance. In this statement is contained the complete description of every movement." This was a grand generalisation of an infinity of particular phenomena. Well, in a small way there is an element of the same impressiveness in the work of an algebraic formula.

The philosophical interest of an algebraic formula may not appeal to every one; but there still remains the usefulness of a formula. No one who has to make any practical use of mathematics in life can dispense with formulae. A formula is compressed information. To calculate the tax on a motor car, the horse-power needed to propel a ship, the range of a projectile under actual conditions, the strength of a girder—whatever the problem may be, there is a formula waiting to tell us all about it. A formula economises thought. No doubt the use of formulae to economise thought is open to abuse in teaching; we do not always advise boys to economise thought. A wise teacher will have nothing to do with formulae in the earliest stages of a subject, *e.g.* in teaching mechanics. But the fact remains that in real life mathematical formulae are almost as necessary as the penny post. And teaching that does not bring out this point is missing the whole *raison d'être* of algebra teaching.

Another fundamental idea is that of functionality. For the sake of non-mathematical hearers (if there are any present), let me give the instance of the bicycle-pump. Put your finger over the nozzle so that no air can escape; and then try to pump. You find that you can push the piston in, compressing the air inside. The further you push, the greater becomes the resistance. There is a mathematical relation between the force you exert and the distance you can push the piston; to push 1 inch you must exert a certain force; to push 2 inches you must exert so much more force (not necessarily twice as much), and so on. We say that the force is a function of the distance. The force and the distance are called variables; the one variable is a function of the other. Given the force you can calculate the distance pushed; given the distance you can calculate the force. This relation can be expressed as an equation connecting the two variables. Call one of them x and the other y, and you have an equation connecting x and y. Furthermore, you can make all this visible to the eye. You can draw a curve, a graph as it is called, exhibiting to the eye the way in which the pressure increases with the distance.

This graph is simply a visible form of the equation, just as the written word is a visible form of the spoken word.

Whenever one measurable thing depends on another measurable thing, you have a case of functionality; you have an equation and you have a graph. To the mathematical eye, life is full of functions and graphs.

Again, the idea of continuously changing quantity enters as a fresh idea. Arithmetic does not present this idea; nor, in fact, does algebra, as algebra was taught in my schooldays. But the idea should come into algebra, *via* graphs and variation; it is bound up with the elementary idea of functionality. I lay stress on the word "elementary"; for, of course, the pupil may learn later that the ideas of continuity and functionality are not essentially associated together.

Then there is the idea of solving a problem by an equation. This is a big step forward from the use of purely arithmetical methods. It is a new idea to a boy, and, at first, till he gets tired of solving equations, rather a fascinating idea. He gets at two different expressions for the same quantity by looking at the problem in two different ways; he clinches the matter by equating these two expressions; and the rules of algebra will finish off the problem for him.

These are some of the new ideas that algebra offers. I think that such ideas should enter into every liberal education. How far boys realise these ideas, even dimly, in our present system of teaching, is doubtful to me; I am not sure that they are not obscured by the number of sums that are set. We insist on the technique of algebra so much that I fear we lose the spirit.

Please do not mistake my meaning; I am far from recommending that boys should be fed on ideas alone. There must be a certain amount of stodgy routine work. They must do work for the ideas. We do not want an education resembling Falstaff's mess-bill: "But one halfpennyworth of bread to this intolerable deal of sack." What I fear is that at present they get an intolerable deal of bread, and no sack at all.

We have not given enough thought to the place of algebra in the education of different classes of boys and girls.

A few girls and a greater number of boys will have to use mathematics in later life. I suppose that hardly any girls are in this class except future mathematical teachers. The boys who will use mathematics are the engineers, architects, officers of the Services, teachers, and some others. For all these the technique is essential, and is not usually overdone. It is astonishing how helpless a boy is in using algebra who has not been through a long and stern drill.

But the majority of boys and the great majority of girls will not make any direct use of algebra in later life. Algebra should

be taught to them for the sake of the ideas, not for the sake of specialised skill and technique. At present all boys and girls are taught in much the same way, polishing up a tool that only a fraction of them will use. I should say that half the time spent on algebra is an educational waste for the ordinary pupil. And this must remain the case till examinations change. If my proposals were put into force with examinations as they are, the examiners would call the results deplorable; and so they would be.

I am trying, you will notice, to establish a distinction between the *educational* study of algebra and the *technical* study of algebra. The methods of instruction imposed nowadays by examinations I should describe as erring on the technical side for the majority of pupils. The stress they lay on manipulation is justifiable if a pupil is to make use of mathematics in his career; but cannot be justified if mathematics is, for him, merely a branch of liberal education.

These children who have a turn for algebra have no difficulty with the technique. I would let them go ahead and follow their bent irrespective of plans in life; all my remarks have applied to the average child who has no special bent for mathematics.

Perhaps I shall be charged with talking heresy, and I shall have to admit the charge. I do not know that any representative body has recommended this discrimination between different classes of pupils in algebra teaching: I have to ask you to consider my views for what they are worth. If these views are right in theory, then in the long run organisations and examinations will amend themselves accordingly. We are not here to frame practical compromises with examining bodies; we are considering what is an ideal scheme of education. I cannot help believing that teachers have gone on too long in an absent-minded way teaching algebra for the sake of algebra. We have to think how to teach algebra for the sake of those we teach; and we have to ascertain the educational value of each way of teaching it.

I shall be asked to define more precisely the distinction I draw between educational and technical algebra teaching.

Perhaps it is unwise to answer this question; some people who might agree with me on broad principles may fall foul of me if I come to details. I do not even know whether there will be enough agreement on principles to make it worth while to draw up a detailed scheme.

But I am bound, even at this stage, to go some short distance to meet the demand for more precise definition.

Let me first explain what I do *not* propose. I do not propose to classify whole broad divisions of the subject as technical and cut them out of the "educational" scheme.

Take the case of quadratic equations. It would be a mistake to cut out quadratic equations from the general curriculum. They

introduce new ideas; the idea that an equation, a problem, can have more than one solution; the idea of coincident roots; the connection of factors with roots; the graphical illustration.

Now all these points can be brought out if we limit ourselves to quadratics readily soluble by factors. I would therefore regard the solution of quadratics by factors as suitable for the general or "educational" course.

When we go further and proceed to the class of quadratics that cannot be solved readily by factors, but need the method of "completing the square," every teacher will realize that we have taken a step involving much more considerable technical difficulty. A great amount of drill is needed to give facility in solving the general quadratic. The only new principle introduced is the possibility of irrational and imaginary roots; an important fact—but for elementary purposes it is sufficient if the master works a few cases of those types, with the class, on the board. The possibility of imaginary roots is not, indeed, shown up by the ordinary type of quadratic set in examinations, for the examples chosen have to come out with real roots.

I suggest, then, that, for educational purposes, we may limit ourselves to quadratics readily soluble by factors, giving enough practice to ensure facility with this type; and that the large amount of additional practice needed for facility in solving the general quadratic should be classed as technical work, and as such is out of place in "general education."

Again, take the case of algebraic fractions. I would not cut out algebraic fractions from the educational course; I would classify them. Boys will tell you that they seem to be always doing fractions. We want to lighten this burden of mechanical perfection. I suppose that the instructive idea that the average boy should gain from studying fractions is this—that he may generalize the operations he has learnt in arithmetic. Would he not gain the idea if we were content to demand facility in operations with a limited type of fractions—say fractions with numerical denominators, or fractions with monomial algebraic denominators—some clear limitation that could be applied to examinations? If we require of a boy certainty in dealing with the general type of fractions, is it not technical skill that we are demanding?

I trust that I have made it clear that I should expect a certain degree of manipulation skill from all boys. No doubt there is a disciplinary value in the neat methodical working needed for success in algebra. This has to be taken into account, but it does not help us to decide where to stop. I propose to determine the necessary degree of dexterity as follows.

When a boy begins a new subject in algebra, he must not be obstructed by inability to perform the elementary operations

needed. For instance, before he begins quadratics he must have mastered the art of factoring. We have to present one difficulty at a time; in taking in a new idea a boy has quite enough to do without having the additional trouble of applying an unfamiliar rule.

In effect, some pieces of manipulation are *stepping-stones.* We ought to look through the course and decide which are the stepping-stones. We shall find that a good many pieces of manipulation generally taught are not stepping-stones for the boy of average requirements. They lead somewhere, but he has no need to go to that place. For example, the alternate division process for H.C.F. leads somewhere; it leads to a certain process in the higher study of theory of equations. But the average boy does not want to step in that direction; all the factors that he will need can be found by inspection.

This paper would be incomplete if I did not indicate a way of providing for this dual course of algebra, the educational and the technical, within the compass of one school. There is generally some sort of bifurcation near the age of 16; boys make up their mind, in some sort of way, what they are going to do in the way of specializing later in life. Up to this point I would have all the algebra teaching of the educational type. After this point there is generally a difference in the amount of time given to mathematics; some do more, some less. For those who do less I would have the algebra still of the educational type, and the examination ahead of them ought to conform to the same tpye. Those who do more are making a technical study of mathematics, because they like it or because they need it. They must give some part of their extra time to grinding up their tools. There must be a good deal of manipulation in those classes. But the manipulation will take less time at this stage, for two reasons, the boys are older, and again, the classes are not clogged with other boys who have no use for this sort of work.

When I propose to cut down the time spent in algebra drill I do not mean to suggest that less time should be given to mathematics. Probably this is not excessive in most schools. But is there not a possibility of using the time so saved to open the boy's eyes to some of the further developments of mathematics—trigonometry or mechanics or even calculus? At present we set him to sharpen a tool—algebraic analysis—that the class of boy I have in mind is likely never to use. Cannot we use our mathematical time in a broader way—trying to give the average student, as he grows older, an outlook, a glimpse of the vast possibilities inherent in the subject?

THE DISCUSSION.

Having delivered his address, **Mr. C. Godfrey** said that if the proposals which it embodied should strike a sympathetic note in the meeting, he hoped it might be moved that they be referred to the Committee.

In opening the discussion, **Prof. Alfred Lodge** said: I think it is most necessary that Mr. Godfrey's suggestions should be put before the Teaching Committee. I do not think we should all agree, perhaps, with the lines of demarcation which he draws between the different ways of teaching pupils. As a matter of fact, I suppose the teaching in the early stages would be pretty much the same with all. Distinction would be necessary only with the senior classes. If I may speak of boys merely by way of illustration, Mr. Godfrey's desire would seem to be to teach the philosophy of algebra to the classical-minded boys without taking a great range or insisting on great technical skill, while proceeding to the further range of mathematics and to drill in skilful manipulation with those who are going to develop mathematically.

It seems that Mr. Godfrey would exclude imaginaries from the philosophical treatment, but I am of opinion that those who are going to take the philosophy of the subject should be equally as interested in imaginaries as in real quantities; and I should like to suggest that we should be wise not to lay down a hard and fast rule that certain boys are to be taken thus far in their studies, while others are to be set a different limit. We may proceed by different methods, but we do not necessarily reach different limits.

It would be good, I think, both for those who are intending to use their mathematics in practical work and for those who are studying the philosophy of the subject if stress were laid on the underlying principles. For instance, in explaining the method of obtaining the H.C.F. of two expressions, there are certain cases in which the pupil cannot readily get the factors when dealing with each expression separately; but if you have a couple of expressions, and take the underlying principle that the sum or difference of any multiples of them must contain the common factor, and use this principle explicitly, the boys will understand the process, and the result will be better than if they were required to blindly go through the formal method of successive divisions. They will thus be ready to appreciate this method properly when they do come to it.

With regard to fractions, I do not agree with Mr. Godfrey that we get the philosophy of algebra by dealing with fractions that have only numerical denominators. Surely the treatment of fractions having algebraic denominators is just as much a part of the philosophy as the treatment of those having simpler

denominators, such as 2, 3, or 4. In fact, I do not think the boys are learning the philosophy of algebra unless they are taught to realise that algebraic denominators are dealt with in exactly the same way as simple numerical denominators. The understanding of this is a part of the training in the philosophy of the subject.

It seems very desirable that Mr. Godfrey's suggestions should be put before the Committee for its careful consideration, but they would need to be elaborated in greater detail.

Dr. F. S. Macaulay: It seems to me that Mr. Godfrey has given us a disquisition on mathematics in general rather than on algebra in particular. The principles which he advocates are those which should apply to the teaching of all mathematics. It may be that his special reference to algebra is due to its being the most difficult subject to teach, and because, perhaps, there is more difference of opinion, or rather lack of opinion, in connection with the teaching of it than with any other branch of mathematics. I do not see how the Committee can deal with any reference to them on the subject of algebra alone. As far as I can see at present, the only thing they could do would be to issue a few recommendations to the effect that more attention should be given to the teaching of principles in mathematics than to the mere working of examples, with the special object of being able to cover more ground than has been covered in recent years; because it seems to me that the teaching of mathematics tends to become not broader but more restricted, on account of our giving so much attention to the details.

I believe that there has been an improvement since the issue of the reports by the Mathematical Association. We certainly cover more ground than was covered six years ago, but we do less work in the same time than our predecessors of twenty years ago.

Mr. Godfrey wisely advised us to leave out all work that did not form a stepping-stone. But, so far as the bookwork is concerned, it is difficult to point out anything which is not a stepping-stone. Mr. Godfrey gave us as an example the H.C.F., but this, in my opinion, is a very important stepping-stone, not merely for theory of equations, but for higher algebra. I agree, however, that H.C.F. should be banished from the first course of algebra, which is all that the ordinary boy accomplishes. No boy ought to be given difficult examples. Our teaching should be confined to easy ones, so that the boy may be able to make greater progress towards new ideas. The objects which I have always tried to put before myself in methods of teaching, and especially in teaching algebra, is to give the boys neither too many nor too difficult examples, and to give them from time to time short lectures on the principles of the subject.

Mr. Godfrey seemed to infer that we never talk about principles

at all. I should hardly think that is true of many teachers. Surely the aim of most teachers must be to impart to their pupils the principles of algebra, as well as the power of manipulating the subject. I was surprised, too, to hear that he seemed to think the principles were easier than the manipulation, and I cannot agree with him there. I think the principles are more interesting. But it requires a higher type of mind to understand principles than to acquire the mere ability to work out examples.

Mr. P. Abbott: The views put forward by Mr. Godfrey constitute, I think, a counsel of perfection. It seems to me that the one essential difficulty is that of organization; a difficulty which he dismissed somewhat lightly, but which is, after all, at the root of the matter. It is not so difficult, perhaps, in the case of schools of the type to which Mr. Godfrey seems to refer, where, as he says, there is a certain division taking place among the boys of about fifteen years of age, but in the majority of the secondary schools in the country there is no such division. Many boys will remain at a secondary school until they reach the age of sixteen, and then they will either finish their school career altogether or begin to specialize. I do not know whether Mr. Godfrey would confine this division to the teaching of algebra alone. It seems to me that it should not be confined to algebra, but should apply equally to the other branches of mathematics if at all. Does Mr. Godfrey think this would be possible in schools of the character I have indicated? As a matter of fact, such a course would probably lead to specialization at an early age; and it is this desire for specialization which all true educationists are fighting at the present moment. Certainly a great deal of what is suggested by Mr. Godfrey is now carried out in practice. Even in the case of those pupils to whom technique is afterwards of importance, for example engineering students, with whom specialization is begun rather early, the technique is often subordinated to the educational aspect, but the danger is that specialization is frequently begun too soon, and the technique predominates at too early an age.

I am convinced that such students would prove better mathematicians afterwards, so far as their technical work is concerned, if the foundations were laid more broadly and less stress laid on technique and "practical mathematics" in the early stages. At the same time, I would point out that the technique is far from being uninteresting to boys. If the technical aspect is used judiciously, boys find a certain amount of pleasure in the manipulatory part of their work. As Dr. Macaulay has pointed out, the theoretical part and the study of fundamental principles are much more difficult to the child than the mere technical part.

We all agree with Mr. Godfrey's principles; but when we come

to the consideration of details we are likely to fall out. I think that one of the details to which he referred will very well bear out my remark. I allude in particular to the question of the solution of quadratics. It seemed to me that Mr. Godfrey was inconsistent at this point. He laid down, as one of the fundamental principles of algebra teaching, the importance of generalization; but when he comes to quadratics, he banishes the idea of generalization, and says that he teaches his pupils merely that type of quadratics which may be dealt with by a special method. It is, in my opinion, desirable to make a beginning with the general method of solving any quadratic, after which we can introduce, as a particular method, the use of factors. We are handicapped in this matter by the text-books, which frequently lead the student to imagine that all quadratics can be solved by simple factorization. We have all come across pupils who have abandoned certain quadratics when they found their results "would not come out," as they put it.

It is very difficult, so shortly after hearing the paper read, to formulate one's thoughts on this exceedingly important matter, and for that reason alone I think it would be well if Mr. Godfrey's requests were referred to the Committee.

Dr. T. P. Nunn: I should like to welcome Mr. Godfrey's most interesting paper very warmly, and to join with other speakers in suggesting that his proposals should be submitted to the Committee for consideration. I am not sure that Mr. Godfrey was well advised in withholding from us his distinction between the "educational" and the "technical" aspects of algebra teaching. He has driven us to make our own interpretations, and these may be wide of the mark. Thus, while I think that I am in agreement with Mr. Godfrey's views, I should hardly be so if, as Mr. Lodge suggests, by the "ideas" he meant the philosophy of algebra. Mr. Godfrey has brought before us a problem of curriculum, and such problems can be solved only by reference to the nature and order of succession of the motives in the pupil to which at different stages of his development the teacher can appeal. I believe that we can specify with reasonable certainty the normal course of the motives which naturally determine a boy's (and no doubt a girl's) activities in science and mathematics. In the earliest stage attention is drawn to objects novel and attractive in themselves, and is sustained only so long as they continue to exercise these charms. As Plato told us long ago, Science is born of Wonder. This stage is not prominent in the development of the mathematical interest, but is no doubt represented by the child's pleasure in the relations of numbers, magic squares, the act of counting up to large numbers, and the like. In the second stage this naive pleasure in numerical relations and processes ceases to be the central motive to mathematical activity,

and becomes subordinated to the motive of utility. By this I do not mean that the boy of thirteen is anxious to store up useful knowledge to meet the possible eventualities of adult life. I mean, to take an example, that the thing that most interests him with regard to similar right-angled triangles is the possibility of using them to find the height of a tree in the playground. In the third stage, the centre of interest shifts from the utility of mathematical concepts and processes to the concepts and processes themselves. The rigidity of a proof and the sufficiency of a definition now become the natural subjects of discussion. It is here that the "philosophy" of mathematics—as I understand the term—has its place. In the utilitarian stage we are contented to accept an irrational number as a symbol for (say) the length of the hypothenuse of a right-angled triangle calculated to any suitable degree of approximation. In the last stage, we are concerned to find a definition of the cardinal numbers that shall be applicable to the irrational equally with the rational numbers, and we are so concerned not because any practical consequences depend upon our success, but merely in the interests of the clearness and completeness of our ideas.

I venture to suggest that Mr. Godfrey's problem belongs to the second of the three stages I have named, and that the "ideas" which are to form the subject-matter of instruction in algebra between the ages of 11 and 15 should, in the main, be selected in accordance with the principle of utility. That is to say, they should be ideas which enable the boy to deal more effectively with the problems which confront him in the physical laboratory, upon field expeditions, etc.; or they should give him some insight into the activities proper to the great constructive and scientific occupations; or, lastly, they should make intelligible to him those matters of general concern which engross the attention of the demographer, the economist, and the actuary.

Miss B. Hewett: One speaker said: The question, so far as it affects girls, is rather different; but from their point of view I very much welcome Mr. Godfrey's paper, and agree with nearly all the suggestions he has made. They bear upon the work I have personally tried to do for a good many years: this work has been hindered by the type of examination paper set, from the old days of the terrible South Kensington papers to the London Matriculation papers of to-day, which demand a vast amount of calculation in a time allowance which is far too short, while the use of logarithms is forbidden. (Hear, hear.) Fortunately, I have not been bound by examinations, but many schools must be.

I think Mr. Godfrey is right when he says that excessive manipulation is deadening to the pupil, though I have found that the mathematical girls take pleasure in doing this while the

principles are being explained to the others; but I do not think, as some speakers seem to imply, that girls should be allowed to be satisfied with the pleasure they find in being able to get the right answer to difficult mechanical operations; we need to make them *think*, and that is the difficulty. I expect a great many of those present have had to deal with pupils coming from elementary schools who are able to do arithmetic with extraordinary facility, and they have often been surprised to see the way in which these children work at sums which they do not in the least understand. Some of us, I dare say, have read a story lately which contains a profound truth. A boy defined algebraic symbols as "Things we use when we do not know what we are talking about." (Laughter.)

Mr. W. N. Roseveare: May I make a few remarks as one who does not quite agree with Mr. Godfrey? It appears to me that algebra should not be treated in the way he seems to suggest, as if it were a separate subject. Algebra means to me the whole language of mathematics, the whole science of symbols, including trigonometry—everything, in fact, except geometry.

I think that in our joint discussion to-day we should take the position that the duty of the mathematician is to furnish accurate proofs, while science should supply the information which comes from rough and ready solutions. If our boys are unable to do algebra, how are they going to solve problems connected with the parabola, for instance? With regard to not teaching the square method of solving quadratics; as to surds, I think we all agree that in quadratics these should be worked out to so many places of decimals. But we want the square method for dealing with any quadratic expressions that may turn up; for instance, for ascertaining the maximum and minimum values in graphs and for finding when a projectile is at a given height. I very much fear that if we are content with graph-work as opposed to strict algebra, we shall have given up our duty so far as it is complementary to the work of the science teacher. It seems to me that our special work in school is to provide accurate, and therefore to a great extent algebraical, proofs of what the science teacher can only do by graphs and rough and ready methods.

Mr. J. V. H. Coates: I do not agree that the matter before us is of sufficient importance to require reference to the Committee. Nothing we have heard amounts to more than an indication that we should emphasize the educational aspect of the mathematics we are teaching, and that there is a danger of attaching too much importance to mere technical ability. It seems to me that all the papers and discussions we now have embody that view, and that while we, as teachers, have special regard to the educational value of our work, we recognise, of course, that there must

also be the technical aspect. Take, for instance, the example we have just had in reference to the general method of solving a quadratic equation. It has been clearly shown that this method has a strong educational value, and certainly I should choose it in preference to the method of factors, because that has practically no educational value: it is purely technical. It seems to me that the discussion generally has indicated that the difference in our views depends upon the particular part of algebra which we regard as having the greatest educational value, and that while it has been shown that every part has a certain value, emphasis should be laid on that aspect of the method we adopt which is most educational. But we must not work at the expense of the technical aspect of the subject. In other words, the problems that we set must be of a practical character. Equations of a particular type must not be set. I think we are all agreed on these points, and I do not think there is any matter which the Committee need discuss.

Mr. E. H. Butt: Last year I spoke rather strongly on the question of the influence of examinations upon our work. I am rather pleased to think that, in the case of the London University Examinations, for which we have to prepare our pupils, there has been a little change made. The change bears upon the side of the intelligence of the work, but the examiners still hold to a terrible heresy which is deadening in its effect. If you take the problems, one after the other, you will see that they are literary puzzles. (Laughter.) Well, that is not mathematics. Years ago I used to delight in teaching for the Junior and Senior Cambridge Examinations, and though the syllabus was much more extended, the work was a real pleasure. One could get the boys on. But now we are kept down and deadened by these terrible literary problems which have nothing to do with mathematics. There is nothing broadening in the whole work. I am rather pleased to be able to speak of the growing recognition of the intelligent side of the work, although the examiners still exact a terrible amount of technical ability.

With regard to the need of putting the matter into the hands of the Committee, I agree with most of our members that something should be done in that way. Mathematics is a delightful subject, and we want it to be so. We can make the boys feel this, and I am quite sure, from my experience of a long life of teaching mathematics, that pupils, who never seemed to have any mathematics in them, suddenly develop a power for them, and begin to feel that there is something in the beauty and the manipulation of mathematics. I do not mean that we should keep a boy down to difficult work. There is nothing in that. If a boy is to go to a technical college, where he must prepare certain definite results, he must be technically correct, but the

educational value of his work is of a very low standard. There is a beauty in all the branches of mathematics, particularly in algebra, that pays for teaching. I have found a delight in teaching them, and I think that boys, and those who know anything about me, realize a perfect delight in working at the philosophical part of their studies, and also in a good deal of the hard technical work. But there must always be a little elasticity given by our examiners, who now dictate to us the kind of sums that we shall do, and also the difficulty of those sums. We do want that elasticity. As Prof. Perry said: "There is no worse enemy to education than examiners." This is the second time in my life that I have spoken much about examiners, but I hope that some change will take place which will lead not only to a certain amount of pleasure in our own work, but also in the lives of the boys and girls whom we teach. If the Committee can do anything in that way, I hope they will not fail to do it.

Mr. C. H. Richards: I should like to make a suggestion as to a line upon which the Committee might consider this question. Might they not take into consideration the advisability of preparing a few sets of specimen papers which should embody more or less the views that have been put forward by Mr. Godfrey?

One speaker remarked that the H.C.F. was about the only thing which could be reasonably left out of elementary algebra. Personally, I do not see very much use in long division, and I think a great deal of time is wasted upon it. At any rate, it certainly might be postponed to a comparatively late date. Several speakers who have mentioned quadratic equations have spoken as though it was undesirable to begin solving equations by the use of factors. I fail to see why they should ever be solved by any other process. It seems to me that in solving a quadratic, when factors cannot be found by inspection, the line to take is to express the equation as the difference of two squares equated to zero.

With regard to the question of fractions, I cannot say that I quite agree with Mr. Godfrey that fractions might be limited to the extent which he suggests, because of the distinctions which should be drawn between arithmetical and algebraical fractions. For example, a proper fraction in algebra is not the same thing as a proper fraction in arithmetic.

The Chairman, at this point, said: We seem to be getting into details which perhaps the Committee might discuss, but there is one line of agreement running through all the speeches. Dr. Macaulay said he thought the proper thing to do is to limit the difficulty of examination. In a sense, that is exactly what Mr. Godfrey proposed, and so, I think, it will be found that other speakers are not so much opposed to one another as they thought. I suggest, therefore, that there is a very good case that the

Committee should work upon a question in which this meeting has shown so great an interest.

As it was now one o'clock, the Chairman decided to close the discussion; and the consideration of Mr. Godfrey's paper was referred to the Committee on the Teaching of Elementary Mathematics.

The other papers presented were given very briefly, including:

"Elliptic Trammels and Fagnano Points," by Prof. P. J. Harding.

"Alternatives to Euclid's Parallel Postulates," by Mr. T. J. Garstang.

"The Geometric Interpretation of Homographic Equations and their application to Loci," by the Rev. J. J. Milne.

"Involution," by Mr. C. V. Durell.

Mr. W. J. Dobbs exhibited his "Patent Inexpensive Balance." The makers are Messrs. Dobbs Bros. of Wolverhampton.

On the motion of Mr. Garstang, seconded by Mr. Dobbs, the question of "The use of the Notion of Direction with regard to Parallels" was referred to the Committee on the Teaching of Elementary Mathematics.

JOINT MEETING OF THE MATHEMATICAL ASSOCIATION AND THE PUBLIC SCHOOL SCIENCE MASTERS' ASSOCIATION.

THE Joint Meeting was held at Westminster School on Wednesday, 12th January, 1910, at 3 p.m., Professor A. R. Forsyth, Sc.D., F.R.S., being in the chair. The subject of discussion was the Report of the Joint Committee of the two Associations on the "Coordination of the Teaching of Science and Mathematics."

The Chairman: As all present here this afternoon are aware, the immediate purposes of our gathering are the reception and the consideration of a Report framed by a Joint Committee, which is composed of members of both the Associations and of a couple of members nominated by the Association of Head Masters of Preparatory Schools. The Report now presented deals with matters of substantial importance, especially to the curriculum of our secondary schools. It has been published and has been circulated; I shall therefore assume that we may take it as read.

So far as my own part in to-day's proceedings is concerned, my position (so I am told) is that of an impartial person: yet, only so far impartial that an early resolution was passed to exclude, from the chair to-day, the actual president of either Association. In whatever character you regard me, you will allow me to thank the Joint Committee for the honour, gladly accepted, which they have done me in asking me to preside.

And, to let me pass at once from things solely personal and entirely foreign to the discussion, you will let me appeal to your indulgence, if these introductory remarks of mine should only inadequately assist the cause that appeals to you all from motives which, because of their varied ambitions, may stir your sympathies in varied cogency.

Your Joint Committee, on the one hand, is obviously influential in its composition: on the other hand, it is sanely representative of men who are thoughtful and skilled in the range of subjects that had to be discussed. The Report which they have produced shows that they have been steadily diligent in their labours. And an outsider (meaning me, if you like) may suspect—though his academic suspicions need not work to its detriment in the least—that the Report is the final result of much accommodating discussion: perhaps, even, it may include occasional minor compromises about isolated details which are not essential to its value. The central governing fact before us to-day is the formulation of a constructive plan, aiming at the achievement of a project of supreme importance to all who are directly concerned with subjects that are acknowledged to stand in the front line of secondary education in our country. You will let me, at this stage, express not merely my hope, but also my belief (a belief which rests upon the terms of a letter I have lately received from Australia, even though the letter deals with only a cognate subject), that concerted action, on the part of your two Associations, may exercise an influence far beyond the confines of what, in customary phrase, I have called "our country." It is, of course, possible to treat a large education problem so that its solution shall satisfy the immediate needs of a large set of schools, when their external conditions are more or less the same. Thus if final school examinations, held after all teaching has come to its end, are the same or are similar, and if they dominate the situation (as, too often, they do unhappily dominate it, under that inverted estimate of the relative importance of teaching and of examination which is tolerated among us), the solution of the propounded problem becomes thereby so much the simpler. But, as I read the Report, it is not thus restricted. It is based upon such experience of teaching as is forthcoming from careful and even meticulous enquiry. Its recommendations, while guided by the information that has been collected with sedulous care, are not limited solely to the range of attainable information. It has ideas, as well as facts, in its composition. Its aim is nothing less than the correlation of mathematical teaching and of science teaching in schools. Its aspiration, so far as aspiration can be realised into achievement, is co-operation between teachers of mathematics and teachers of physical science over such ranges of their subjects which, while belonging to both of them in a very

real sense, have too often in the past been regarded as rather distinct branches of a liberal education.

Yet, while noting the bearing of so important an issue, noting also the range of the Report as well as the detailed character of its recommendations, you will further note that the Committee in their large task have, with a wisdom that may be commended as prudent, kept within the terms of reference. A casual and hasty reading of the Report, made without reference either to the terms of their appointment or to Reports published by each of the two Associations which relate to them separately and specially, might have led the casual and hasty reader to a wrong impression as to the ideals which inspire the Report. So you will further note their preamble—let me recall a phrase that I have often heard, "Preambles do not matter save (possibly) as explaining, but not as establishing, the effect of a document embodying recommendations": and let me quote some sentences, indicating the tenour of the Report, from the prefaced preamble:

"The Committee wish it to be understood that they are not laying down a scheme for the teaching of mathematics, or for the teaching of science. Each of these subjects has its special functions in a balanced education, and requires its special methods of presentation. To discuss these functions and methods is not within the scope of a Joint Committee such as the present; but forms rather the field of activity appropriate to the two separate organisations represented.

"The two subjects, however, have numerous points of contact, and there is a certain debatable ground between them which the present enquiries show to be in need of survey by a Joint Committee. On the one hand, the Committee find that energy is wasted by unnecessary overlapping, and on the other there is need for increased sympathy and co-operation between teachers of the two subjects.

"The recommendations contained in this Report are directed to the solutions of [these] problems."

So we are not concerned with the whole field of mathematics or the whole field of physical science, in so far as they occupy areas of mental discipline of secondary education.

In the presence of such a Report, it is natural for each of us to return upon his own experience, if for no other purpose than that of criticism and appreciation. My own experience goes back to youthful school days when I had an excellent drill (not in physics, which the Report recommends as the initial subject, but) in inorganic chemistry, both theoretical and practical. Later, as an undergraduate, I had to learn things, which probably now many school-boys know practically; my acquisition of them was entirely theoretical in those now rather distant days. Recalling an experience which was fairly normal, I now think

that our knowledge was too often acquired in different ranges with a labour and an occasional ineffectiveness all the more oppressive, because there was no indication of the relations which were an intrinsic part of their common existence, and which could have found definition, expression, and explanation. Moreover, in my later student days, there was a sad lack of relation between the mathematical side and the experimental or observational side of one and the same physical subject.

At such a time as this and on such an occasion, there is an irresistible temptation to become reminiscent. Let me, for an ulterior purpose of illustration, mention only a couple of recollections of my own. I remember how, when one of my teachers (and he was a very great teacher) discoursed about the floatation of bodies in a perfect fluid, he summed his results into a golden rule: "Remove the fluid, and put a force acting upwards." The rule was clear enough in its statement and its directions; but I fear that the suggestions, as to the procedure which it enjoined, may have had some share in killing any real interest that might have arisen in me about the subject of hydrostatics. The other recollection relates to geometrical optics where, among other things, we were taught or had to learn about telescopes. The picture in the book made the astronomical telescope look like a medicine bottle: it was directed at a small realistic arrow of nearly its own breadth, placed at a distance of nearly its own length. But I had never seen the actual thing: and while compulsion was put upon me to become expert with appropriate formulæ, there was no compulsion to make me become acquainted with the actual thing. I plead guilty to any charge of youthful indolence in not going to look at an astronomical telescope on my own account: I might retort that the humble opera-glass was then not even mentioned in connection with the text-book account of Galileo's telescope.

These facts have been stated, not to give some light relief to rather heavy remarks, but to illustrate an opinion which I hold somewhat firmly. I consider that, when real physical things are made the subject of mathematical teaching, it is both desirable and necessary that there should have been or should be some actual observation of the real physical things, perhaps preliminary to the mathematics, perhaps more or less concurrent with the mathematics. Further, I think it will be all to the good that a mathematical master, when required to teach an applied subject (such as mechanics), should have some direct observational knowledge of the physical phenomena and their relations as well as the mathematical knowledge which allows him to submit them to calculations, whether simple or recondite. The formal developments in directions, which can diverge widely, belong to later stages; for the moment, I am concerned with the beginnings

which, though of an elementary character, can serve to provide the useful outlook upon reality in teaching such a subject.

Thus far I have spoken of what seems to me a spirit of desirable improvement in the teaching given by all mathematical masters: and it is the merest justice to point out that the Report shows how considerable and how progressive an advance has been made in recent years. But is there nothing to be said, reciprocally, about the teaching given by schoolmasters in physical sciences? is there nothing to be said about the utilisation of possible calculations, whether purely numerical or even occasionally more formally mathematical, in connection with experimental methods and with experiments? Probably there is much to be said; but, as I am not a person who can say it with any right of experience, you will let me shelter myself behind the authority of your Committee. When present at experimental lectures, I have often recalled to my own mind (and I mention it now in palliation of my boldness in speaking at all to science teachers) the substance of one remark made by Lord Rayleigh in what was his first lecture at Cambridge after he had been appointed Cavendish Professor of Experimental Physics. He declared that quite good results could often be obtained from simple and even apparently clumsy bits of apparatus; and then he warned (or advised) his hearers that no experiment should be completely parted with, until every scrap of information, that could be extracted from it by observation and comparison and thought, had been extracted. Such has been the economical practice of the greatest physicists: the advice may now, thirty years after it fell upon my ears, seem platitudinous to those of you who have expert knowledge: but it is none the less sage. But you may ask, in silent wonder, what is the bearing of Lord Rayleigh's remark in connection with the subject before us? It is absolutely simple. Just as mathematical teachers of a physical subject should have full regard to the phenomena of their subject when they deal with their mathematics, so it would be an advantage if science teachers, when building up their subject by progressive experiments, should make the significance of an experiment, or of a set of experiments, as effective as possible by giving to it, or to them, a numerical (or even a mathematical) interpretation whenever this can reasonably be done. Another platitude, you may think: yes, but I do not want to get out of my depth; so this will be the limit of my remarks on this topic, hoping that, neither in word nor in spirit, they are hostile to anything in the Report which is submitted to us to-day.

What are the inferences to be deduced from all these criticisms, veiled under the form of recollections, and these suggestions, disguised under the form of personal opinions? To my mind one inference is that, perhaps for various groups of boys, and certainly

for selected portions of some subjects of teaching, there is much ground that is common to mathematics and physical science within school ranges. But, as is so often the case when ground is common, there can be public and private contentions about rights, and duties, and limitations. When this happens in the matter of any school subject, in such a way that there is overlapping, without agreement or even without understanding on the part of the teachers, energy may be wasted. High aims can then drift apart. There is no exaggeration in saying that, in and under the traditions of half a century ago, mathematical teachers were often disposed to pay too slight a regard to the active observation of the phenomena of a physical subject; and not a few of them have been inclined to act upon the opinion, placed on printed record by a mathematician of great eminence: "It may be said that the fact [in an experiment] makes a stronger impression on the boy through the medium of his sight, that he believes it the more confidently. I say that this ought not to be the case. If he does not believe the statements of his tutor—probably a clergyman of mature knowledge, recognised ability, and blameless character—his suspicion is irrational and manifests a want of the power of appreciating evidence." We have passed from that position.

Is there no criticism, perhaps less stern in spirit just because it is less informed in fact, to be levelled at the teaching of physical science? You must judge mainly for yourselves, because your knowledge of present practice goes far, very far, beyond any knowledge to which I can pretend. In leaving the final judgment to you, let me give you another quotation. You all know how the work of Faraday inspired the work of Maxwell; and perhaps know something of the pleasure given, in those older days, to that chief among physicists by the growing achievements of the then young mathematician. You know also how the combined work of those two great men has stimulated an ever-growing army of pioneers. Most of you, perhaps all of you, will know that Faraday had no formal mathematical training at any period in his life: and you know (it is the last demand to be made upon your knowledge, which I assume to be not less extensive than the knowledge of Macaulay's typical school-boy) that Faraday shows, in his published *Experimental Researches*, one of the most powerful mathematical minds that have ever found uncanonical expression in unconscious (was it unconscious?) mathematical language, notwithstanding his lack of methodical training and of formal discipline in mathematics. Yet there is upon record one piece of advice, given to Tyndall by Faraday, which seems to me not without some possible import here to-day. It may have sprung out of his matured experience: it may have been dictated by what he saw of the sympathetic work of the then younger

men, such as William Thomson and James Clerk Maxwell: from whatever source he was impelled to give the advice, he asked Tyndall to work out any experimental result as far as possible, "so that the mathematicians may be able to take it up." It is, of course, too much to expect that Faraday's characteristic and systematic treatment of every experimental result should be followed by all physicists. Some, as I am prepared to believe, always make the attempt: some (am I very suspicious?) rarely try: and some physicists make the attempt sometimes. Am I too optimistic in thinking that, if some of the science teaching were controlled, perhaps it is in these days, by the spirit of Faraday's advice to Tyndall, much good would accrue, to the training of school-boys, if scientific results could be so left that mathematicians, even mathematical school-boys, could "take them up"?

In these latest remarks I may have gone too far afield. Yet, in spite of their seeming irrelevance and superfluity, they are dictated by a wish for more cohesive action between two sets of workers who, so far from being antagonistic to one another, should be sympathetic in desire and in action. As I understand the present situation in your deliberations, your Joint Committee has framed a scheme which is intended to secure (so far as any scheme can secure) co-operation between two groups of teachers. The scheme also propounds a programme of work that fairly is common to their subjects, even in their respectively larger aspects. Further, it is based upon experience already attained; it thus has some claim that, though only now to be offered for your acceptance, it has already undergone some critical tests. It may require modification from time to time: was there ever a detailed scheme, however good, the working of which did not occasionally require modification? Manifestly, care and thought have been unstintedly devoted to its construction. And my own hope, in all sympathetic acceptance of its recommendations, is that, if it is generally adopted by all of you so far as your power may go, if it is worked with active goodwill, yet with an observation at once critical and friendly, its promises of good results, in the blending of the interests of mathematics and the interests of physical science, may be fulfilled. Above all—and this I believe to be your main object, whatever other purpose may be attained or may be foiled—it promises improvement in the range of that part of secondary education with which you are unitedly concerned.

Now let me wander away again, briefly, from the immediate context of the Report, mainly for the purpose of showing that the aims of your Committee are aims which are not confined to secondary schools alone. I wish to refer—in no controversial spirit, and certainly without any revival of controversial issues—

to refer only to matters upon which there was general agreement, when changes were adopted not long ago in Cambridge about the Mathematical Tripos. The advocates of change undoubtedly desired change for the sake of the students who looked forward to mathematics as the subject (or possibly one of the subjects) of their profession in life. But they also desired change, if only it could be conveniently and harmoniously adopted within the range of the Mathematical Tripos, for students to whom a mathematical course would be of real value, yet who looked forward to physics or to engineering as the subject of their profession in life. What, then, did the advocates of change suggest as regards the only examination about which they had any power to report? Let me read a few of the regulations, submitted to the Senate and adopted (as I think, without any silent reserve of challenge) by the Senate: they refer to Part I. (which, normally, can be taken at the end of the first year or at the end of the second year of residence):

3. The questions in the physical subjects of the Schedule shall be of such a character as to test knowledge of the physical phenomena and their relations, and not merely an ability to deal with the analytical developments of the subjects.

4. A large proportion of such riders as are set shall consist of simple examples illustrating numerically or otherwise the corresponding theory.

8. . . . In order to obtain Honours in the examination, a candidate must satisfy the examiners both in Pure Mathematics and in Applied Mathematics.

And the first two of these (with only one verbal alteration that is unessential to us who are here) are regulations also for the Part II., the most advanced mathematical examination held by the University.

These regulations are not quoted solely because they are indications of settled thought at Cambridge: I need not disclaim any desire to impose Cambridge thought upon others who, whether within or without that University, also desire, not merely to improve mathematical training, but also to improve science training, by giving to any subject what any other can bestow as useful. I have quoted them mainly to show, by even only one illustration, that your Joint Committee is far from being a small Committee of prophets crying in a wilderness.

At this stage, an extension of any amount could be given to my remarks by an abstract (which might be anything just short of complete verbal reproduction) of the Report. It is better that the document should be read, and studied, and digested by each man for himself: and rather than reproduce it, in a summary hash, I would invite you to fix your attention upon the

uses to which it can be turned, allowing it to appeal to you in all its strength, and to face you even with all its points of attack.

For the Report must have all sorts of points where it can be attacked. Whatever care has been given to its construction, it is compact of detailed suggestion: and, therefore, it is open. to detailed criticism. Those, who have the responsibility and have devoted labour for its construction, will doubtless give full attention to criticisms from every quarter, especially to criticisms which are based upon experience of its working hereafter. My own hope is that those teachers, who already have made it active, and those who will have opportunity of putting it upon its trial, will exercise a little liberty in the sequence of its recommendations, so that, without introducing undue variety, a teacher may be allowed to exercise his own individuality if he wishes to give it play. Above all, I wish to think that the Report, while dictated by a common belief, is neither a creed nor (remembering the work done two and a half centuries ago in a chapel of the ancient abbey under whose shadow we now meet) a Westminster Confession of Faith and Catechisms. Rather it is a syllabus, to be worked and to be tested, to be criticised and perhaps changed as experience, upon which it is based, may require it to be changed in the light of greater knowledge.

Let me beg you to be patient in the quite early years of growth: all experience shows that patience is needed to gather and to glean full harvests from even the most careful husbandry. Let me beg you, also, to be cautious, not distractedly rushing forward in eagerness: yet not to be too cautious, through timidity or reluctance. To all of you whom it may concern, let your supposedly impartial chairman quote an Italian proverb, "Che va piano, va sano: che va sano, va lontano," which, being translated, may read, "Go gently, and you will go safely: go safely, and you will go far."

And so my remarks will be brought to an end. They may not have risen to the ideals of your Committee. Easily could it be understood that they should have failed, either in exposition of the issues involved, or in expression of the motives of the constructive spirits who have given their best to you. In that event, let the blame be imputed to me alone. In any event, let me beg you to give to the Report the most careful consideration in your power: alike upon its intrinsic merits, upon the probable range of its influence, and (most of all) upon its possibilities in the achievement of purposes which, as I hope, you all desire with a full and willing mind. And if, in a last sentence, you will permit me to say so, your careful consideration of the Report is the least of the tributes to which the work of your Committee is entitled. (Applause.)

The Chairman then called upon Mr. C. Godfrey and Mr. D. Berridge to address the meeting upon the Report from the respective standpoints of the two Associations.

Mr. C. Godfrey: I do not think there is any strong reason why I should trouble you for more than a few minutes. Perhaps the chief reason why I should show myself is in order that any criticisms or hecklings which this meeting may wish to direct against the Report may have some visible objective. The Joint-Committee appointed by the two Associations was not in a position to refer this Report to the two Associations for confirmation. That would not have been a practicable course to adopt, and the Report therefore is presented without such support. It is presented on its own merits, and it must not be taken as authorized by either Association.

I think I need not say more than one or two words—those, perhaps, of an historical nature. I suppose that 20 or 30 years ago there would have been no demand for a Report such as that we have before us. In those days I fancy there were few points of contact between the teaching of mathematics and the teaching of science. The teaching of mathematics was then, perhaps, more rigidly deductive than it is now. Nowadays the teaching of mathematics includes deduction as an essential portion, but we have found room within its scope for reasoning also of an experimental and inductive nature. On the other hand, I take it that in those days the teaching of science mainly meant the teaching of chemistry. I suppose that the teaching of physics is of more recent growth. Since that time mathematical teaching has developed in one direction, and the teaching of science has advanced in another, and has grown to meet the teaching of mathematics. The two spheres of influence have expanded, have touched, and have now overlapped. The fact of the overlapping of the two spheres appears to me to necessitate some delimitation of frontier. Such a delimitation is a matter of detail, and therefore rather a dull business, I fear; but it is not for me to apologize for the type of recommendations put forward.

This overlapping of the two spheres, then, is one reason which made the Report desirable. Another reason was the feeling that there was room for more mutual interest and sympathy between the teachers of the two subjects. (Applause.) It may not be fitting for me to speak of the teachers of physics, because my own experience has lain rather in the direction of mathematics; but if one may speak with all caution, the way in which teachers of science might develop their interest in their colleagues' work, is by endeavouring to appreciate more fully the aims of mathematical teaching in the training of the mind. It is natural that teachers of science should be rather more concerned with the

power of mathematics as an instrument for getting out results (and that is undoubtedly an important part of the scope of mathematical teaching) but, on the other hand, mathematicians will always be concerned rather with the function of their subject as a mental gymnastic training, and I think it may be hoped that physicists will increasingly enter into their point of view in this matter.

Passing on to the other side of the question, I feel that we mathematicians may do a good deal to interest ourselves in, and to understand the aims of our science colleagues. Mathematics is divided into 'pure' and 'applied'; in pure mathematics the main thing is mathematics; in applied mathematics the main thing is application. I mean to say that pure mathematics, so far as it is taught not for research but for educational purposes, is largely concerned with mental discipline. But when you come to consider applied mathematics, I feel it is quite essential that the teacher should constantly have an eye to the real concrete problems, to the real objective to which this mathematics is applied. I think there has been a danger in the past that the teaching of applied mathematics should be too much on the lines which I have suggested as appropriate to the teaching of pure mathematics. The teacher should, in my opinion, be more concerned with the investigation of real phenomena to which the mathematics are applied, rather than with the invention and solution of questions for which the science in the question is a mere excuse. I trust that the recommendations of this Report will tend to put that matter in a more real and living aspect.

I hope that criticisms will be freely offered. I cannot promise that the Committee will be in a position to take any account of them, because as far as I understand, the Committee ceases to exist at this meeting; but if there are criticisms which, in the opinion of the Associations concerned, necessitate a re-consideration of the Report, perhaps the time will come for that later. (Applause.)

Mr. Berridge: I am quite sure that the great majority of those present feel that they know what we who are on the Joint-Committee think, and are brimming over with expectation to hear those whose opinions they have not yet been told. (Laughter.) I wish, however, to mention two points in order to ease the discussion later on. First, there is a point connected with the statement in the Report as to the present condition of mathematical and science teaching in our schools. I should like to remind you that in the circular which we sent out there were between 70 and 80 questions, and that we received replies from nearly 300 schools, consequently the number of questions answered is very large. We do not claim infallibility, but we

think the Committee have a right to suggest that they have laid before you an accurate representation of what is the present practice.

It has been suggested that we have been rather unfair to the Committee of the British Association, which issued a report last year. We have mentioned, on page 11 of our Report, that there is a rather serious disagreement between the results obtained by ourselves and those obtained by the British Association. I have written privately on this matter to Mr. Daniell, but I am anxious publicly to emphasise that this disagreement relates to one point only, viz., the order in which chemistry and physics are taught. Our investigations, which were very considerable, show that in 34 per cent. of Grammar schools chemistry is taught before physics, and I am sure that all science masters here will agree that this is not the right order in which the subjects should be taken. Although on this point the results we have obtained are in disagreement with those obtained by the British Association, on all others our results are identical with theirs: when we remember how much wider our own investigation has been, we cannot but admire the excellent manner in which the British Association must have set to work in making the selection of schools to which they sent their applications.

I now wish to speak to the mathematicians present, and to throw myself on their liberality. Included in the replies which came to me as the Secretary were at least half-a-dozen abusive letters. I have been a schoolmaster for nearly twenty years, and during that time I have interviewed a large number of parents, and therefore I can say that I do not object to abuse (laughter); but the abuse which I received was almost entirely confined to my having asked on the circular: "Are the boys at your school taught the use of the slide-rule?" If these letters represent the general feeling among mathematicians, I want to explain why the scientists particularly ask that the slide-rule should be introduced early. We want to make a boy think, and our work in the laboratories is almost always directed to giving a boy some observation to make and then asking him to deduce some fact from the observation. We find we get much better work from a quantitative than from a qualitative experiment, but an ordinary simple measurement in a laboratory generally means a good deal of calculation before we can use it in the class-room afterwards. To take the simple case of measuring the gas evolved when a given weight of metal dissolves in an acid; the calculation involves two separate sums in multiplication and two in division, you will find that the average boy takes about seven or eight minutes for the calculation, and the chances are that in the end he will not have got a correct

arithmetical result. We ask, Is all this Arithmetic worth the trouble? It is not science; you mathematicians would be the very first to say it is not mathematics. The main question therefore is not whether we shall use slide rules: we say emphatically we must; but, since the slide-rules are to be in the hands of boys, is the science master (who is usually a very hard-worked animal) hurriedly to explain their use to the boys, or is that task to be left to his mathematical colleague, perhaps in connection with the Theory of Indices and the use of Logarithms? The latter course would confer an incalculable benefit on your science colleagues. Is the slide-rule to be explained in the mathematical class-room, or is it to be used only in the laboratories? I should be very glad if gentlemen who in subsequent discussion object to the use of the slide-rule will keep in mind the explanation I have given as to why we want it.

Prof. H. H. Turner, in moving the following resolution:

> "That this meeting is in sympathy with the attempt embodied in the Report to correlate more closely the teaching of mathematics and science,"

said: You will notice that this resolution is of a very general kind. It is only a resolution of such a character that can be dealt with by a meeting of this magnitude. The bulk of the work of detail must be done by your committee, and the admirable report before us has already involved a great deal of work on their part. My intention in moving this resolution is to ask for criticisms in detail upon the report, though it is clear that there is a convenience in keeping the actual resolution which initiates the discussion in very general terms. I am myself in hearty sympathy with this movement for the closer correlation of the teaching of mathematics and science. I suppose, as representing the mathematicians, it behoves me to speak more or less from a mathematical point of view; and from that point of view I welcome it, especially as an expansion of the interests of mathematicians.

To my colleagues of this morning, I ventured to express the opinion that we have been too narrow in our interests hitherto. When other people are present, perhaps I had better not dwell too much on that point. At the same time, I think it is an undoubted fact that we have fallen short of what is expected of us. May I give an instance in support of my remark? Some years ago the Royal Society had to nominate a member of the governing-body of Winchester, and they nominated an eminent mathematician. I was addressed by one of his colleagues on the governing-body soon afterwards, with an expression of regret that my friend had not been at a certain

meeting that afternoon, because a scientific question was before them on which his advice might have been useful. I inquired what the matter was, and was told that it was a question of the drains in the school. (Laughter.) Such was his idea of what a mathematician ought to know.

We have heard before of the limitations of the mathematician, and especially of his want of knowledge in practical work. I welcome this report, therefore, as offering one way in which the interests of the mathematician may be expanded. Let me admit at once, that if hastily read, it may seem as though the mathematician were asking a little too much. Thus in Recommendation No. 3 it is carefully laid down that the mathematical master shall teach the measurements suggested in Recommendation No. 2. Again, in Recommendation No. 10 it is fixed that Statics be taken in the upper part of the school as part of the mathematical teaching. It is only in Recommendation No. 8 that we see what is to be done by the mathematical staff and what by the science staff. This may look as though the mathematician were looking too sharply after his own interests in these matters; but if you will regard his attitude from the point of view that he is trying to educate himself in the best possible way, I think the recommendations will assume a different complexion.

I do not think the mathematician will be likely to wish to meddle with chemistry or other matters which are remote from his sphere. He is only too conscious of the practical difficulties. May I add another personal reminiscence? Some 30 years ago an eminent Second Wrangler in my school-days was asked by the headmaster to teach chemistry. The headmaster of those days was a man who not only invited one to do a thing: he saw that it was done. One day I was working in the physical laboratory, when from the next room, in which this master had been teaching chemistry, he erupted rather violently in a state of collapse. He had with great triumph made some chlorine; then, in his excitement, he forgot in which of the jars the chlorine was, and attempted to find out by smelling them, with the result that his lungs were not in working order for some considerable time. But those days of crude experiments in the combined teaching of mathematics and science may be considered as belonging to the past: we hope now for a more organised combination, and it is in this hope that I commend the resolution to your good will. (Applause.)

Professor H. E. Armstrong, in seconding the Resolution, said: I have much pleasure, on behalf of the Association of Science Masters, in seconding the very mild Resolution which is before the meeting. Looking round the room and seeing so many active, eager faces, I cannot help thinking that there are many

amongst you who are quite prepared, not only to be constructive but even to be aggressive. (Laughter.) I think this is a body which cannot model its action on the lines of the Headmasters' Association and confine itself to the expression of pious opinions; it should be prepared not only to advocate but to take action. We have had enough talking about matters of this kind; we want to do something. (Hear, hear.) I, on my side, welcome this movement very heartily, and I think I may say, without fear of contradiction, that all the science masters will welcome it, because they are really the people whom it will help, particularly in the direction of having that granted to them for which they have been begging and praying for many years. Of course, on our part, this is no new request; for at least 20 years we have been asking for the absolutely essential assistance of the mathematician at the commencement of the course. What we have had to complain of was and what we complain of still, is, that when boys are sent up for science work, they have not received the preliminary instruction in mathematics which will enable them to do what is required of them. This applies even to simple things. The difficulty would be met if the recommendations of this Report should be carried into operation.

Many believe that those who will benefit most by the projected change in attitude on the part of teachers in schools will be the mathematicians themselves. I cannot help thinking that masters will find the subject deeply interesting to them when it is made practical; much more interesting than under the present conditions. As to the boys, there is no question whatever what the effect will be. It is well known to those who have had to do with the teaching of boys who have left school and have come to college, that it rarely happens that mathematics has been treated as a real subject; we all find the greatest difficulty in getting them to apply their mathematics to practical work. If this co-operation between mathematicians and teachers of science in schools be secured, that sort of difficulty should cease to exist.

I think Mr. Berridge said something with regard to the necessity of teaching physics before chemistry, and in the Report, I think, something is said with regard to the need of teaching Heat at an early stage. I do not think that matters. What we have to remember is, that, in the first instance, we must start our chemical teaching by dealing with properties of materials as such; for that purpose we have to characterize them from a physical point of view. That is the important consideration.

I do not think it will be necessary for me to do more than formally second Professor Turner's proposition and to say how

heartily I am in sympathy with it. I think all the science masters will be with me in that view. (Applause.)

The Chairman: Before submitting for discussion the motion which has now been moved and seconded, I understand that Mr. G. F. C. Searle is prepared to make a few remarks.

Mr. G. F. C. Searle: I thank you for the opportunity which you have given me for introducing a break into this discussion. I merely want to refer to the course in practical physics held at the Cavendish Laboratory in Cambridge during the Long Vacation (August). Last year we were able to do a little more in the way of organising than was possible in the previous year, and the class was, I think, considerably more successful. The experiments done by those masters who came to Cambridge are mentioned on page 14 of the Report. Professor Sir Joseph Thomson has given permission for the course to be held again, and I shall be very glad indeed if any masters who are interested in that work at Cambridge will speak to me afterwards in the Exhibition, in order that their suggestions may assist us to make the course as useful as possible.

The Cavendish Laboratory has, in some ways, very large resources, and if sufficient notice were given to us beforehand, many experiments in addition to those done last year might be quite easily performed. My main object in speaking now, is to say that I shall be very glad if any of you who feel interested would speak to me, and perhaps afterwards send me lists of the experiments which you think it would be profitable to do, so that we in Cambridge may have the opportunity of considering them and of seeing what can be done to make the course this year as successful as possible.

I am very heartily in sympathy with this attempt to make the relation between the teaching of mathematics and the teaching of physics in schools as close as possible. I have given practical proof of that. In Cambridge, with the help of my colleague Mr. Bedford, we have organized a set of lectures, in experimental optics, for the candidates for the Mathematical Tripos; so that those who are now taking that tripos will have seen some practical work, and will perhaps be a little more prepared for teaching science than the older mathematicians have been.

Mr. S. De Brath: As a preparatory schoolmaster who has worked for twelve years at the correlation of physical and mathematical teaching, based on the previous training and practice of an engineer, I am glad of your permission to say a few words on this matter. It was a sense of the need for such correlation which led me into the scholastic profession, and a short experience as an assistant master convinced me that the reason for the very imperfect grip on realities of the average boy, lay much more with the pre-

paratory school than with the public school. "What is the use of algebra and geometry and all the stuff we have to learn unless you are going to be a schoolmaster?" a boy of fifteen said to me at the time when I was thinking of taking up the profession. After he had been shown how to use triangles to find distances, the principle of the range-finder, and that forces being proportional to lines all kinds of practical results flow from the properties of triangles, he astonished his master on his return to school by his complete change in comprehension and output: from a notorious idler he became a willing pupil. He was an example of the effect of a little correlation.

The interests of the practical man are not those of the specialist. He wants to be able to appreciate the work of specialists and apply it, but he does not want much pure mathematics or much pure science, though he does want to think scientifically, *i.e.* with exactitude; and he reproaches his schoolmasters if they do not lead him to this.

At the present day physical processes enter so largely into so many departments of production that the demand for this kind of knowledge extends far beyond the engineering profession, and is as indispensable as humanist culture to men who are to play their parts worthily in the English nation, hard pressed as it is by organized science in the art of peace and war.

The chief purpose of teaching physics in schools I take to be the training of young minds to look to proximate and effective causes; the chief purpose of mathematics to train them in exactitude and clear thinking—to connect the "How?" with the "How much?" These two habits make the scientific mind, which is above all things desirous of seeing truly and acting decisively.

Inasmuch as abstract knowledge has historically been preceded by the concrete knowledge from which it was abstracted, and from the time of Archimedes to that of Newton physical problems have stimulated mathematical knowledge, I welcome Resolutions 1-13. But as a preparatory schoolmaster I protest against Resolution 14:

(1) On grounds of the liberty of action for preparatory schools;

(2) On grounds of educational theory and practical experience.

The business of a preparatory school is radically different from that of a public school, because of a psychological fact.

Up to an age which may be put at fifteen, the mind of the average boy works inductively and not deductively, and the method of teaching should match this fact. When it is so matched the success is notable. The great difficulty of pure mathematics to the average boy lies in the highly abstract

nature of its expression. Many never attach any real meaning at all to this expression: and this is the great difficulty of the mathematical masters at the public schools.

But the habit of attaching full meaning to their mathematics is, in my experience, only reached when a boy has been accustomed from the very beginning of his arithmetical work to discover (heuristically) from concrete cases a "rule" or generalization, *e.g.* to make his own tables of length, area, volume, weight and money from concrete objects, and then to apply the rules so reached to concrete problems. Similarly in algebra, if boys first construct their equations by discovering that the product of the weight into the arm on either side of a steelyard is an identity, before solving for any unknown quantities, they at once obtain an insight which no explanations can give.

In geometry, if they discover the properties of plane figures from instances on which they have to generalize, the facts remain with them without wearying and interminable repetition. This is not a Utopian theory, but a practical fact.

This is my idea of Primary Correlation, and those who do me the honour of taking a copy of the printed notes which I have prepared rather hastily for this meeting will see the way in which it is practically carried out.

Physics should similarly be taught in preparatory schools in a practical manner: (1) to describe objects by their colour, shape, size, weight, texture, hardness, etc.; (2) to observe the elementary properties of water and air; (3) to realize the equilibrium of forces; (4) to observe the effects of heat.

Whereas in the mathematical class the physical is always subordinated to the mathematical treatment and made the stepping-stone to a conclusion of pure mathematics, in the physics class the aim is to familiarize with unseen forces, and to show by very simple means that they are capable of measurement. In this way a boy applies his physics to his mathematics and *vice versa.*

Such courses are preliminary to the more complete treatment in the public school; the boy who has gone through them does not come to the latter quite strange to the phenomena. I shall not venture to tell this distinguished audience how I think this training should be carried on, but this I do say—that if the preparatory schools sent up boys really accustomed to use their mathematics, having done most of the work of appendix D, in a rough way, either in mathematical or in physics class, you gentlemen would find them much easier to teach, and would be able to bring them to a standard as good as any in Europe without cram or over-pressure. They are not up to that standard now, nor anywhere near it.

If this commends itself to you, the practical means of giving

effect to it are ready to your hands. You need only induce the H.M. Conference to modify the common examination for entrance to the public schools so as to demand from all the boys the practical power of using their mathematics pre-supposed by Resolutions 1-13, and to offer Science Scholarships based on a sound elementary knowledge of Hydrostatics and Heat as Clifton does; and to these I should like to add Elementary Mechanics.

But I would beg you not to use the weight of your authority to limit the activities of those preparatory schools which desire to show the educational value of correlated science.

Mr. C. S. Jackson: I think that the existence of this Report is more important than its contents. Not only was it unnecessary to adopt the Queensberry rules, but the discussions of the committee were throughout of the most cordial character, and gave, I think, great promise of good results being ultimately obtained. An important question to consider to-day is this, "What is the origin of that feeling of anxiety which is undoubtedly felt by some experienced and competent teachers of mathematics as to the outcome of those tendencies which may be ticketed "practical." I put aside altogether the severe logician who draws a sharp distinction between learning and teaching. He learned mathematics when he was at the University, and is under the impression that since then he has been teaching it. (Laughter.) But putting him altogether out of the case, there is still this anxiety felt by some very able teachers, and so we ought to ask ourselves what is its origin, and to inquire what justification it may have.

I think there are three main points. Such a teacher regards mathematics, quite apart from any question of utility, as the one great source of precision of ideas, and of power to realize the force of simplicity and to avoid a superfluous redundance of tautological platitudes. Secondly, he attaches importance to the maxim that you ought to teach, that you can only teach one thing at a time. It is quite true that an abstract principle must be approached through a concrete instance, but to try and deal with a mathematical and a physical difficulty simultaneously is almost certain to produce disaster. To reconcile in detail the principle of one thing at a time with the principle of beginning with a physical instance will give us all occupation for a long time.

I think there is also a feeling that there is something in the notion of thoroughness. To be able to solve a quadratic equation is one thing. To be able to solve a quadratic equation with easy factors, or with a bit of luck, is not the same thing. Besides these main points, there is the sense also of doubt as to the utility of methods—a feeling that the method may be good in the hands of its originator, but that perhaps it is not quite so good in the

hands of its adopter. Professor Maitland, who is not famous as a great teacher because of his far greater fame as a lawyer and historian, said: "Our Anglo-Saxon ancestors did not arrive at the alphabet by easy stages." Can their descendants arrive at it by the heuristic method?

I think that teachers of mathematics may fairly say that these are considerations which should give us pause; but I do also ask, Is there not something on the other side? Are we satisfied with the mathematical state of the average boy when he leaves school? I make no apology for again quoting a sentence or two from the preface of a German book recently reviewed in the *Gazette*. The author says: "In every country there are numbers of boys who leave a secondary school with a little arithmetic, a little algebra, a little geometry; and this little they soon forget, for it never formed a connected whole in their minds." Now, I am afraid that is quite true. The author goes on to say (and again one can confirm his statement by experience). "A few years later the boy wishes he knew more mathematics." We have all heard this said. We are all in the position of having problems submitted to us with the touching confidence that we shall be able to do something with them. The infantry officer designing a range-finder—the horseman interested in the dynamics of the proper seat for jumping a hurdle—the doctor studying the gait of a paralysed person, or the dynamical justification for swinging one's arms when walking—the amateur boat-builder—the billiard player—the pigeon fancier, all start to propound their riddle by saying "I wish I had done more mathematics at school."

And now I ask, Cannot we do just a little more for the average boy, and get over the ground a little faster. A great deal of the stuff that is called mathematics ought not to be so termed. Mr. Gheury in an interesting paper [*Gazette*, vol. iv. p. 92] said: "The mathematician will set his class to plot $y=\sqrt{\cdot 5-\frac{1}{10\sqrt{x}}}$." That isn't mathematics. In this respectable locality I won't venture to find a word to describe it. (Laughter.) Our mathematics may be useful, and science work will help to make it useful; or it may be elegant, to use the good technical word. Let us lay down the principle that if it is neither useful nor elegant we won't have it. One service this Report may do for us. Experimental work will save us from beginning at the wrong end—with an abstraction. The students must begin with the facts. Then, I think, we shall appreciate that the soul of mathematics is generalization; but the greatest curse of mathematical teaching is premature generalization. (Applause.)

Mr. T. J. Garstang: I should like to make one or two remarks with regard to the possibility of extending the basis of the

Report. One of them is relevant to a remark that has already been made. We all desire, I think, to see our pupils progress more quickly, but we want them at the same time to retain, from the mathematical point of view, some element of logic. I feel encouraged to submit a question which I hope Prof. Forsyth will touch upon in any reply that he may be disposed to make. It is this: Mr. Jackson has reminded us of the difficulty under present conditions of acquiring early knowledge heuristically; it must be thrust upon our pupils. Would it not be a good thing therefore if we were to try and take as the basis for mathematical teaching, a line more in conformity with that which is taken for granted in physics? The adoption of such a plan would enable us to push the subject on a little further. We should then be able to start with the same foundation in mathematics as we now start with in physics; and having taken for granted and formulated a sufficient number of postulates as a firm foundation, I think we could proceed more quickly, and indeed more logically.

I cannot help thinking that we mathematical teachers are still compelled by the examinations to spend too much time in proving theorems to our pupils in the beginning of their course, especially in Geometry; and that if we could give some purpose to practical measurements by being allowed to formulate the results of Practical Geometry in a sufficient number of postulates, the boys would feel that they had not wasted time, and we should get along more quickly in mathematics, and be better able to meet the demands of the physics teacher than we can at present. That is one point. It is not necessary for me, I think, to give any details, because Prof. Forsyth will, I am quite sure, understand to what I refer; moreover, the exact details could be referred to the Mathematical Association.

The other point has reference to Logarithms. I do not know whether the Secretary of the Joint-Committee wishes distinctly that the slide-rule should be used, or whether he prefers Logarithms. I draw a distinction, and for this reason: I find that pupils are able to learn the use of Logarithms before they are able to take sufficient care of a slide-rule that is within the bounds of sufficient accuracy. It may be an unfortunate personal experience that my pupils are more destructive than others, but I should like a definite statement on the point I have raised.

Mr. Berridge replied that he was a "whole-hogger."

Mr. Garstang (resuming): I do not know whether I am definitely against the compulsory introduction of the slide-rule, though my experience leads me in that direction.*

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* A slide-rule requires to be used constantly in order to be of real value; pupils, who use the slide-rule regularly in physical work, naturally acquire facility in its use; but it has not been shewn that the slide-rule is a normal instrument for those pupils who

There is just one matter in which I wish the Science Masters' Association would join with the Mathematical Association. I speak of the need of removing the bounds set by certain syllabuses in the examinations. It is not of much use for mathematical masters to spend a long time in teaching their pupils the use and the theory of Logarithms, and get their learning fairly skilfully applied, so long as the pupils have to sit at examinations where Logarithms are not used and sometimes are even prohibited. (Hear, hear.) If the science masters will combine with us here, we shall be able and willing to co-operate most heartily, to the great advantage of all.

Mr. G. F. Daniell: I wish warmly to support the resolution. I heartily welcome this report on the correlation of the teaching of science and mathematics. The essential thing appears to me to be, that the mathematical teacher and the *physics* teacher should clearly define their respective aims, and that each should understand the other. It is only from that standpoint that you will get a really true correlation. I believe that this report will help materially to bring about a discussion between the teachers of these subjects which will lead to a better understanding, and that thus we shall get an effective correlation. At the present moment that is more important than the detailed recommendations of the report.

I wish to take this opportunity of thanking Mr. Berridge for what he said with regard to the British Association Committee, of which I was the secretary. I am gratified to find that the results obtained by the British Association Committee have been so generally confirmed by the more extensive statistics obtained by the Joint Committee. With regard to the one matter of disagreement, on that I still disagree with Mr. Berridge, because I think that the statistics support all the statements which have been made on behalf of the British Association Committee. The difficulty seems to be due to want of discrimination between the beginning of physics and the beginning of practical physics. They are identical in the public schools, but by no means identical in the smaller schools, where very often a course of demonstration lessons on heat precedes practical physics.

I am indebted to Mr. Berridge for the remarks he has made and for the spirit in which he met my objection when I wrote to him.

I should like to endorse what was said by a previous speaker (Mr. De Brath) about clause 14 of the Recommendations. I consider that this is a rather dangerous statement to go out from the societies. We are aware that there are people who are hostile to

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spend most of their science time in biological work. As mathematical classes contain both kinds of pupils, the slide-rule remains a special instrument and has no general application.

the expansion of science teaching, and I am most anxious that we should do nothing to weaken the position of science teaching in preparatory schools. We need very much to strengthen it, but I am afraid that if we are not careful clause 14 will be misinterpreted.

On the report in general may I say that the important feature of it is the recognition of a close relation between the actual handling of bodies, measurements, and so on, and the mathematical work. I wish it had gone a little further. In the 1st Recommendation we find that measurements of length to the decimal scale should be taught as soon as decimal fractions are introduced. I rather wish to put the order the other way about: that you should begin with the objects and the measurements, and obtain from them your mathematical idea. If, for instance, you are teaching the triangle of forces, you should not introduce the crane to illustrate; the crane should be introduced first, and then we should discover the triangle of forces; after which, of course, we can apply it. This idea does go through some of the recommendations quite strongly, but here and there it appears to have been dropped.

But on the whole I warmly welcome this most useful report in connection with one of the most encouraging advances which have been made in recent years. (Applause.)

Mr. A. Vassall: I have very little justification for speaking at this meeting, but perhaps I have had three good excuses for doing so afforded me this afternoon by the closing remarks of our chairman, by a part of Mr. Garstang's speech, and especially by the remarks of the preparatory schoolmaster (Mr. De Brath) who opened the discussion. I am neither physicist nor chemist, nor even mathematician; and I was surprised to find myself given the offer of serving on the Committee which issued this report. I asked several people for an explanation, and was told that I was there in order that when the members began to ask about things that any ordinary sort of fool could not very well understand, I was to start talking and to continue until they changed the subject. (Laughter.) That will perhaps justify my behaviour to some of my colleagues during our meetings.

There were some who desired that the Report should not be placed in the hands of mathematicians not used to the work in it, and with these I am entirely in sympathy. The tendency of Mr. Garstang seems to me to be a desire to hurry matters a little further than the situation is ripe for at present. In the popular electioneering story of the moment, my function seemed to be to apply the brake. The story goes that a lady asked an eminent statesman who had just sat down to take his cup of tea, "What, after all, is the use of the House of Lords?" He did not answer the question immediately, but afterwards poured the tea into the

saucer and pointing to the saucer said, "This is the House of Lords." (Laughter.) I take it that on this Committee I was fulfilling the function of the House of Lords, my duty being to cool down the oratory of the scientist and the mathematician, which may land them in the very real danger of forgetting the boy. Speaking without understanding in either mathematics or physics, I believe that this report does come very near to the thing which has the interest of the boy at heart, and not that of the teacher only. It is within the intelligence of the average boy, of the boy who is intended to do that sort of work, and is not miles beyond him. No effort was spared by the Committee to produce something which was practical, and which it was hoped that mathematicians would be able to adopt, at all events in an experimental way, as was suggested to us by our chairman; but if I rightly understand the proceedings of the Committee, Recommendation 14 needs restating, because both of the speakers who have referred to it have placed upon it an entirely different interpretation from that intended.

I believe that the Committee in general are fully in sympathy with the remarks of the previous speaker (Mr. De Brath), and that they agree that much of what is called mensuration and allied work should be dealt with in preparatory schools. Some years ago I had to undertake an extensive inquiry into the science teaching of preparatory schools, and I found in several cases no science was taught except qualitative analysis and light; formal science of this type we deprecated at preparatory schools.

If my remarks are at fault, perhaps the members of the Committee will correct me. I have thought, however, that my explanation would perhaps set at rest the minds of Mr. De Brath and Mr. Daniell as to Recommendation 14.

Mr. C. Godfrey: I should like to say a word about paragraph 14. May I take the last clause first?

> "Questions should not be set in formal physics or chemistry at the entrance, or entrance scholarship examinations to the Public Schools."

We thought it desirable to recommend this, because if such *are* set in entrance scholarship examinations, preparatory schools are in effect compelled to teach formal physics or chemistry. We did not think it desirable that this compulsion should exist. The Committee clearly contemplated that a certain kind of physics, physical measurements in fact, should be done in preparatory schools; it suggests "That such times might be devoted to practical measurements."

I do not think there is any doubt at all that boys of the age of 12 or 12½ years could, if properly taught, take physics or even chemistry; but the staff of the ordinary preparatory school does

not, I take it, include science men, and to what extent these subjects can be taught with advantage to young boys by those who have no expert knowledge, is, I think, a matter open to grave doubt. A great number of boys have passed through my hands at Osborne, and for the most part they have not learned chemistry or physics before joining. The few who stated that they had already learned something of these subjects did not do any better than those who professed ignorance.

As a matter of organisation it is clearly convenient that all boys coming to a public school should be in a position to start their physics or their chemistry if possible in one class. If preparatory schools are to teach formal physics and chemistry, we shall be adding to the difficulties of organisation at public schools. We shall be inviting just those difficulties which now exist in connection with Greek.

We therefore felt that, taking in view the whole series of preparatory schools, we could not recommend that formal physics and chemistry should be commenced at so early a stage. I do not mean for a moment to assert that they could not be taken with advantage by a skilful and expert teacher.

Dr. T. P. Nunn: There is no doubt that the Committee has acted wisely in adopting a report which will carry with it the great body of teachers interested in this subject, though the inevitable consequence of their moderation is that those who were ready to go a good deal farther find themselves a little disappointed. On reflection, however, such persons will perceive that the utmost to be demanded of such a report as this, is that it shall be helpful as far as it goes, and that it shall not close the road to further progress. There is no doubt that the document before us fulfils the former condition, but I am afraid that in one or two respects it lies open to the charge of not fulfilling the latter. My friend, Mr. Daniell, has already mentioned the point which I have chiefly in mind, but it appears to me to be a matter of so much importance that I venture to call your further attention to it.

In the first Recommendation it is laid down that a boy should be familiar with the manipulation of decimals *before* he undertakes measurements such as those of area. In Recommendation No. 10 we are told very emphatically that the study of mechanics should be postponed until *after* a certain elementary knowledge of trigonometry has been acquired. Finally, it is very definitely suggested at a later point that the boy should have studied logarithms *before* he is taught the use of the slide-rule.

In each of these instances the same form of "correlation" is prescribed: the boy is to acquire a sufficient mastery of his mathematical methods before he applies them to practical work. Now my objection is that there are many teachers of experience

who strongly prefer the opposite mode of procedure. Upon their view a practical problem should be made the occasion for the introduction of a new mathematical method, and the technical development of the method should follow instead of preceding the demonstration of its usefulness. Thus such a teacher might present to his class the problem of area-determination as the motive for working out the technical process of decimal multiplication. At a later stage he might choose the study of the vector properties of forces as the occasion for introducing the idea of a cosine. Finally, he might make the abstract consideration of fractional indices and logarithms arise from the practical use of an instrument of the slide-rule type. The Committee were entirely justified in not pressing upon teachers a mode of procedure which is relatively remote from current practice; on the other hand it seems unfortunate that, so early in a movement which for some time to come must be considered as in its experimental stage, they should appear to pronounce authoritatively against methods that, *prima facie*, are pedagogically sound and are advocated by teachers of such eminence as Prof. Perry.

Mr. Hugh Richardson: I think we must all be grateful to those who have brought out this Report, both on account of its future usefulness in the schools and for the spirit of the University which it transmits to us. There is one point which I think will need attention when we get back to our own schools and meet some of our colleagues who are not so personally interested in the teaching of science as we ourselves are, and who, being mathematicians, are not so well acquainted with the necessity of science teaching as those who are here to-day. The point seems to me to be this; every science master has learned algebra as far as quadratic equations and also the first six books of Euclid, but very few mathematical masters have learned the use of a physical balance or a burette. Roughly, when we get back to our work, we shall find that each sort of master is ignorant of the other's knowledge. I think Mr. Vassall is right when he says that we must expect our mathematical colleagues to go slowly; but I have no hesitation in regard to leadership in the matter, and I am sure that they must try to move on in their own interests. I think that in ten years' time the mathematical master who cannot weigh and measure, and who cannot do his laboratory work, will be as extinct as the dodo. (Laughter.) This is consequently a very serious matter for mathematical masters, and I think their reluctance to accept these notions indicates that they have great financial interests at stake and that therefore we ought to help them.

The problems of the small school are very different from those of the large school. In a large school, and in schools under education committees, things are "correlated"; but in small

schools people occasionally "talk shop." My experience this last term has been with the small school and the single boy, and I have had to deal with questions which came up in laboratory teaching and which my tired head could not answer. And so I sent the boy to his mathematical master for answers to questions which I knew nothing about. You know how slowly a boy carries a message. Start the boy off and you have plenty of time to go and see the mathematical master before he gets there. (Laughter.) So I sent on a postcard to the master, "Can you look up catenaries and hyperbolic cosines, because Blank is coming to you presently?" And the mathematical master says, "I do not know." And so I reply, "If you look in the index of one of your books, you will find it on page so-and-so." And by-and-bye the boy gets round to him and finds that the mathematical master is perfectly clear upon all these things. And so with such questions as, "What is Simpson's 3/8 Rule, and how do you prove it?" "What is the difference between vectors and rotors?" All these things I do not know about myself, and I have to send a card to the mathematical master in each case, and he always receives it before the boy arrives.

We, who represent different schools, shall all have to think what can be done; for the problem in front is the problem of the education of the ordinary mathematical master. I do not refer to anybody here, because here they are all extraordinary. (Laughter.) I think we must try to be tactful, to lead and not to hurry; but when we have dealt with our mathematical colleagues, we have a still more serious problem before us, and that is, what are we to do with headmasters who are neither scientific nor mathematical; though I think it will be enough for us this year if we confine our attention to our mathematical colleagues.

I speak of them with the greatest respect, because of all the mathematical men I have worked with, two were masters who got most efficient knowledge into the minds of their boys, although they themselves were quite opaque to all scientific reasoning. (Laughter.)

Sir Joseph Thomson: I am afraid that I have no detailed knowledge of the Report. I read it some time ago, and was in entire sympathy with it at the time. It seems to me that the question of the closer correlation of mathematics and science is one of very great importance, because at the present time there is a tendency to teach a kind of emasculated physics—a physics without mathematics.

The appliances for teaching practical physics have increased almost out of knowledge in the last 20 years; but my own experience rather leads me to believe that the most important instrument in physics, that is, mathematics, is less regarded now

than formerly. There is a tendency to pander to the demands of the botanist and of the chemist who demand a kind of physics that would not give a headache to a caterpillar. (Laughter.) If we try to set a question which involves a little reasoning, and if we have colleagues who represent other branches of science, we are told that we are playing into the hands of the mathematician, and that the botanist cannot be expected to answer a question of that kind—that what is meant by physics is a description of pieces of apparatus and of various experiments; in short, physics from which all reasoning is eliminated.

Well, I look to this Report for help to remedy that state of things. I also hope that it will increase the interest, and may I say the belief, of applied mathematicians in their own subject? Nothing, I think, is more amusing than to find, when a mathematician comes to the laboratory, the delight and even surprise with which he discovers that formulae upon which he has been working for many years of his life bear some approximation to the truth. (Laughter.) When a Senior Wrangler finds there is some such relation between the distances of an object and its image in a lens, as the formula predicts, his enthusiasm and surprise are unbounded.

My experience with those who have only had a tripos training is, that they would not back any of their formulae for sixpence. What we have to do for those who are to become engineers is to impart to them a confidence in their mathematical results—to make them believe in their mathematics—and that is what would be done, I think, if we began mathematics and physics together in schools at the very earliest opportunity. They ought never to be separated; they ought to march together almost from the very beginning.

I should like to make an appeal to mathematical masters. Is it absolutely necessary that you should teach so very little mathematics as at present? Is this asking too much when it is borne in mind that boys of 18½ and 19 who come up to the University are in nineteen cases out of twenty quite ignorant of the differential calculus? Your curriculum is very full, I know, but surely it does not seem an excessive demand that a boy who must have a mind equal to some sort of mathematical reasoning should, at that age, be able at any rate to use the simple processes of the differential and the integral calculus. (Applause.)

The Resolution was then put to the meeting and carried *nem. con.*

Mr. L. Cumming proposed a vote of thanks to Professor Forsyth for presiding, and the meeting then concluded.

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